DYNAMIC ANALYSIS OF THE S-TYPE FLUID-CONVEYING STEEL PIPE FOR TSEN-WENG RESERVOIR USING FINITE ELEMENT METHOD

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ABSTRACT

About seven years ago, the first author was invited to attend a meeting held by the “South Water Resources Bureau” to discuss the problem about cleaning sediments of the Jsen-Weng Reservoir located at Tainan, Taiwan. At that time, a suggestion to replace “the conventional sluice gate near free water surface” by “the siphon with its inlet near reservoir bottom” (cf. Figure. A1 in Appendix) was presented and approved by all the attendees. Five years later, about two years ago, the first author was invited again to attend a meeting to review the report for a project about the construction of a tunnel for cleaning sediments of the Jsen-Weng Reservoir. The key point is to review the design of a “S-type water-conveying steel pipe” (or simply called “S-pipe”) with its outlet connecting with an 1.2 km tunnel passing through a mountain. The pipe diameter is 10m with maximum draft at its outlet to be $h \approx 35m$, so that the S-pipe will be subjected to the impulses $F_L = F_R \approx 5384$ tons at its inlet and outlet, as well as the resultant centrifugal forces $F_{c1} = F_{c2} \approx 3760$ tons at its two curved parts. Because of being non-collinear, each pair of impulses or centrifugal forces will induce an unbalanced “couple” with total magnitude $C_{total} \approx 216727$ m-ton, furthermore, its vibration due to pulsating fluid flow may lead to the liquefaction of soil contacting with it. Since the above-mentioned problems are neglected in the reviewed report, the first objective of this paper is to use the finite element method (FEM) for studying the quasi-static deflections of some points on the S-pipe due to actions of the “constant” impulses at its inlet and outlet as well as the centrifugal forces at the two curved parts when the flowing velocity becomes a “constant” (e.g., $V \approx 24.25$ m/s) from the initial static condition (with $V = 0$). The next objective is to study the dynamic responses of the S-pipe due to “pulsating” flow with velocity $V(t) = V_0(1 + \delta \cos \omega t)$, where $V_0$ is average velocity, $t$ is time, $\delta$ is pulsating parameter and $\omega$ is pulsating frequency. Numerical results reveal that the S-pipe will be safe if it is subjected to the “constant” impulses and centrifugal forces, however, its safety may be questionable if it is subjected to “pulsating” flow. It is believed that the original conception shown in the Appendix of this paper should be a better choice.

KEYWORDS: Centrifugal Force, Forced Vibration, Free Vibration, Impulse, Pulsating Flow, Soil Liquefaction

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INTRODUCTION

The studies on the dynamic problem of fluid-conveying pipes are more than 50 years (Ashley and Haviland, 1950; Housner, 1952; Long, 1955; Hsu, 1963), thus, the literature concerned is plenty. Since Ibrahim (2010, 2011) has reviewed several hundred references regarding pipes conveying fluids, only some pertinent articles are mentioned in this paper. For the bending vibrations of a fluid-conveying uniform pipe and that of a uniform beam, one of the main differences is that the former has the Coriolis force and the latter does not have. For this reason, even if the inner and outer dampings are equal to zero, the $r$th eigenvalue of a fluid-conveying pipe takes the form of complex number $\lambda_r = \lambda_{r,R} + j\lambda_{r,I}$, where $\lambda_{r,R}$ and $\lambda_{r,I}$ denote the real part and imaginary part of $\lambda_r$, respectively, and $j = \sqrt{-1}$. Through theoretical analysis and model tests, Benjamin (1961a, 1961b) proved that the articulated fluid-conveying pipe has two types of instabilities, buckling and fluttering. The buckling instability appears when $\lambda_{r,R} > 0$ and $\lambda_{r,I} = 0$, while the fluttering instability occurs if $\lambda_{r,R} > 0$ and $\lambda_{r,I} \neq 0$. The last conclusion is the basis for the instability problem of fluid-conveying pipes. After Benjamin (1961a, 1961b), Gregory and Paidoussis (1966a, 1966b) studied the free vibration characteristics of a cantilevered fluid-conveying pipe and presented a “universal” stability curve. Since, for each curve, the “abscissa” is the dimensionless mass ratio $m_f^* = m_f/(m_f + m_p)$ and the “ordinate” is the dimensionless critical velocity $V_{cr}^* = V_{cr} L \sqrt{m_f/(EI)}$, the above-mentioned “universal” stability curve is available for the uniform fluid-conveying pipes with various pipe dimensions and material constants. However, the last statement is correct only for the Euler-Bernoulli pipe with the effects of shear deformation and rotary inertia neglected, and incorrect for the Timoshenko pipes that are dependent on the slenderness ratios. For a fluid-conveying pipe with the pulsating flow velocity $V(t) = V_0 (1 + \delta \cos \omega t)$ with $V_0$ denoting the average velocity, Chen (1971) used the Hsu’s (1963) method to study the influence of pulsating parameter $\delta$ and pulsating frequency $\omega$ on the area of the instability region. Later, Paidoussis and Issid (1974) plotted the Argand diagrams to determine the critical flow velocity $V_{cr}$ of the fluid-conveying pipe. From the existing literature (Paidoussis and Sundararajan, 1975; Paidoussis and Issid, 1976; Paidoussissand Laithier, 1976; Noah and Hopkins, 1980; Chen and Fan, 1987; Aldraihem, 2007; Yu et al., 2014; Wu et al., 2015), one sees that the problems studied in most existing works have something to do with the stability curves, Argand diagrams or area of instability region of the fluid-conveying pipes. It is also seen that most of the fluid-conveying pipes are single-span, and only a few of them are multi-span (Wu and Shih, 2001; Wu et al., 2015). Furthermore, most of the problems are solved with the classical analytical methods, and only a few of them are solved with the finite element method (FEM) (Kohli and Nakra, 1984; To and Healy, 1986; Chen and Fan, 1987; Sreejith et al., 2004; Aldraihem, 2007; Wu et al., 2015; Arguelles and Casanova, 2015).

It is similar to the railways (Wu and Shih, 2000a, 2000b) or ocean structures (Wu and Chang, 1988; Wu and Chen, 2010) that, for a fluid-conveying pipe imbedded in the soil, the interaction between pipe and soil may be modeled by the uniformly distributed springs with stiffness $k_f$ (Lottati and Kornecki, 1986; Djondjorov et al., 2001; Ryu et al., 2004). Since the value of $k_f$ is dependent on the property of soil (Richart et al., 1970) and must be determined by experiments, in this paper, it is provided by the company building the S-pipe. The last pipe is composed of five segments, including two curved (or arc) pipe segments and three straight ones, use of pure curved pipe theory (Chen, 1972, 1973) or pure curved beam theory (Wu and Chiang, 2003, 2004) for solving the dynamic problem of this S-pipe is very difficult, thus, it is
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solved with the conventional FEM (Kohli and Nakra, 1984; Przemieniecki, 1985; To and Healy, 1986; Chen and Fan, 1987; Sreejith et al., 2004; Aldraihem, 2007; Wu et al., 2015; Arguelles and Casanova, 2015) in this paper. Two types of flow velocity are studied: constant velocity $V_o$ and pulsating velocity $V(t) = V_o(1 + \delta \cos \omega t)$. For constant velocity $V_o$, the purpose is to study the “quasi-static” deflections of some points on the S-pipe due to actions of the constant impulses at the inlet and outlet as well as the constant centrifugal forces at the two curved parts. For pulsating velocity, the purpose is to study the dynamic responses of the above-mentioned points on the pipe. Numerical results reveal that the safety of the S-pipe may be questionable if it is subjected to pulsating flow and the original conception (with the S-pipe replaced by the siphon) should be safer, more efficient and more practical.

SKETCH OF THE S-PIPE

A sketch of the S-pipe is shown in Figure 1. It is a double-shell pipe with an inner diameter (of the inner shell) $d_i = 10$ m and outer diameter (of the outer shell) $d_o = 11.66$ m. The thickness of the inner shell is $t_{in} = 0.035$ m and that of the outer shell is $t_{out} = 0.032$ m. The entire pipe is composed of three straight pipe segments and two curved ones. For the last two curved (arc) segments, the radii of curvatures are $R_1 = 20$ m and $R_2 = 30$ m with subtended angles $\theta_1 = 40^\circ$ and $\theta_2 = 40^\circ$, respectively. In Figure 1, along the center line of the pipe, the arc denoted by $BCD$ is the 1st curved pipe segment with $B$ and $D$ denoting its left and right ends, respectively, and $C$ denoting its middle; similarly, the arc denoted by $EFG$ is the 2nd curved pipe segment with $E$ and $G$ denoting its left and right ends, respectively, and $F$ denoting its middle. Among the three straight pipe segments, two of them are at the left inlet denoted by $AB$ and right outlet denoted by $GH$, respectively, the third one is $DE$ with its left end $D$ connecting with the 1st curved pipe segment and its right end $E$ is connecting with the 2nd curved pipe segment. It is noted that the straight pipe segment $DE$ is tangent to the two curved pipe segments at $D$ and $E$, respectively. Furthermore, the horizontal distance between left inlet and right outlet is about $L_h = 56.11$ m and the vertical one is about $L_v = 20$ m, and the corresponding elevations are: EL175 m at the inlet and EL195 m at the outlet. Since the maximum elevation of the free water surface of Tsen-Weng reservoir is EL230 m, the maximum elevation head for the flowing water at the outlet of S-pipe is $h_{max} = 230 - 195 = 35$ m. Therefore, the variation of average flow velocity $V$ for the S-pipe is shown in Table 1, where $g = 9.8$ m/s is the gravity acceleration and $h$ is the water depth (draft) at the center of the pipe outlet.

<table>
<thead>
<tr>
<th>Draft at Pipe Outlet, $h$ (m)</th>
<th>4</th>
<th>5</th>
<th>9</th>
<th>10</th>
<th>16</th>
<th>20</th>
<th>25</th>
<th>30</th>
<th>35</th>
</tr>
</thead>
<tbody>
<tr>
<td>$V = \sqrt{2gh}$ (m/s)</td>
<td>8.85</td>
<td>9.90</td>
<td>13.28</td>
<td>14.00</td>
<td>17.71</td>
<td>19.80</td>
<td>22.14</td>
<td>24.25</td>
<td>26.19</td>
</tr>
<tr>
<td>$V$ (km/h)</td>
<td>31.87</td>
<td>35.64</td>
<td>47.81</td>
<td>50.40</td>
<td>63.75</td>
<td>71.27</td>
<td>79.68</td>
<td>87.29</td>
<td>94.28</td>
</tr>
</tbody>
</table>
FREE VIBRATION ANALYSIS BY FINITE ELEMENT METHOD (FEM)

In this paper, the vibration analysis of the S-pipe is conducted by using the finite element method (FEM). Some key points concerned are introduced as follows.

MATHEMATICAL MODEL

The mathematical model of the S-pipe studied in this paper is shown in Figure 2, in which, the entire pipe is considered as a free-clamped (F-C) curved beam rested on the elastic foundation with \( k_f \) denoting the “uniform-distributed” stiffness between the pipe and the soil.

FINITE ELEMENT MODEL

According to the sketch of the S-pipe shown in Figure 1 and the mathematical model shown in Figure 2, a finite element model composed of 66 two-node pipe elements and 67 nodes is established as shown in Figure 3. From last figure one sees that the entire pipe is divided into 5 segments, among which, the 1\(^{st}\) segment (\( AB \)), 3\(^{rd}\) segment (\( DE \)) and 5\(^{th}\)
segment ($\overline{GH}$) are the straight pipes to be subdivided into 5, 16 and 9 pipe elements (i.e., $n_{e,1}=5$, $n_{e,2}=16$ and $n_{e,3}=9$), respectively; while the 2nd segment ($\overline{BCD}$) and 4th segment ($\overline{EFG}$) are the curved pipes with subtended angles $\theta_1=\theta_2=40^\circ$ to be subdivided into 16 and 20 pipe elements (i.e., $n_{e,2}=16$ and $n_{e,4}=20$), respectively. For convenience, the last two curved pipes are called the 1st and 2nd curved pipe segments in this paper and their radii of curvatures are denoted by $R_1=20$ m and $R_2=30$ m, respectively. It is noted that, in Figure 3, the unit of lengths is meter (m).

**ELEMENTAL PROPERTY MATRICES**

Each pipe element composed of the entire S-pipe shown in Figure 3 is to take the form shown in Figure 4. It is a two-node clamped-clamped (C-C) beam element with each node having three displacements, $u_1-u_3$ and $u_4-u_6$. Among which, $u_1$ and $u_3$ are the longitudinal (or axial) displacements in the $x$-direction, $u_2$ and $u_5$ are the transverse (lateral) displacements in the $y$-direction, and $u_4$ and $u_6$ are the angular displacements (or rotational angles) about the $z$-axis, respectively. The mass matrix $[m]_e$, damping matrix $[c]_e$ and stiffness matrix $[k]_e$ of such a pipe element are given by (Kohli and Nakra, 1984; Przemiańcicki, 1985; To and Healy, 1986; Wu et al., 2015)

\[
[m]_e = \frac{(m_x+m_y+m_{\text{end}})}{420} \\
\begin{bmatrix}
140 & 0 & 156 & 0 & 156 & 0 \\
0 & 156 & 0 & 4\ell & 0 & 4\ell \\
0 & 22\ell & 4\ell^2 & 0 & 140 & 0 \\
70 & 0 & 0 & 140 & 0 & 0 \\
0 & 0 & 4\ell & 0 & 156 & 0 \\
0 & 13\ell & -3\ell^2 & 0 & -22\ell & 4\ell^2 \\
\end{bmatrix}
\]

(1)
\[ [c]_e = \frac{m_j V^2}{30} \begin{bmatrix} 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 6\ell & 30 & -6\ell \\ 0 & -6\ell & 0 & 6\ell & -\ell^2 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 30 & -6\ell & 0 & 6\ell \\ 0 & 6\ell & \ell^2 & 0 & -6\ell & 0 \end{bmatrix} \]  

(2)

\[ [k]_e = [k_b]_e + [k_f]_e + [k_g]_e \]  

(3)

In Equation (3), the symbols \([k_b]_e\), \([k_f]_e\) and \([k_g]_e\) are the bending, elastic-foundation and geometric element stiffness matrices given by

\[
[k_b]_e = \begin{bmatrix} \frac{EA}{\ell} & 0 & 12EI/\ell^3 & 0 \\ 0 & 6EI/\ell^2 & 4EI/\ell & 0 \\ -\frac{EA}{\ell} & 0 & 0 & \frac{EA}{\ell} \\ 0 & -12EI/\ell^3 & -6EI/\ell^2 & 0 & 12EI/\ell^3 \\ 0 & 6EI/\ell^2 & 2EI/\ell & 0 & -6EI/\ell^2 & 4EI/\ell \end{bmatrix} \]  

(4)

\[
[k_f]_e = \frac{k_e}{420} \begin{bmatrix} 0 & 0 & 156 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 22\ell & 4\ell^2 & 0 \\ 0 & 54 & 13\ell & 0 & 156 \\ 0 & -13\ell & -3\ell^2 & 0 & -22\ell & 4\ell^2 \end{bmatrix} \]  

(5)

\[
[k_g]_e = \left( \frac{P}{30\ell} \right) \begin{bmatrix} 30 & 0 & 36 & 0 \\ 0 & 3\ell & 4\ell^2 & 0 \\ -30 & 0 & 0 & 30 \\ 0 & -36 & -3\ell & 0 & 36 \\ 0 & 3\ell & -\ell^2 & 0 & -3\ell & 4\ell^2 \end{bmatrix} \]  

(6)

where

\[ P = P_p + p_f A_f + m_j V^2 \]  

(7)

In the right side of Equation (7), \(P_p\) is the axial load applied on the pipe wall, \(p_f A_f\) is the fluid pressure on the pipe section and \(m_j V^2\) is the centrifugal force on the pipe section due to structure vibrations. Where the compressions are positive (+) and tensions are negative (-). Furthermore, the centrifugal forces induced by the 1\textsuperscript{st} and 2\textsuperscript{nd} curved pipe segments (BCD and EFG) are considered as the external loads on the entire S-pipe and are not included in Equation (7), because each pipe element shown in Figure 3 is replaced by the straight one. The definitions for the other symbols appearing in Eqs. (1)-(7) are as follows: \(m_p\) = pipe mass per unit length (including pipe material and cement in the ballast tanks), \(m_f\) = fluid mass per unit pipe length in the pipe, \(m_{\text{add}}\) = added mass (of water) per unit length surrounding the
pipe, $V = V_0 =$ average flowing velocity of fluid, $\ell =$ length of each pipe element, $E =$ Young modulus of pipe material, $A =$ cross-sectional area of pipe wall, $A_f =$ cross-sectional area of fluid column in the pipe, $I =$ moment of inertia of cross-sectional area $A$, $k_f =$ stiffness of soil for per unit pipe length, and $P_p =$ axial pressure on the pipe wall.

**TRANSFORMATION MATRIX**

From the finite element model shown in Figure 3 one sees that, besides the first five pipe elements ($n_{e,1} = 5$) near the inlet and the final 9 ones ($n_{e,9} = 9$) near the outlet, the local coordinate systems $oxy$ for all the other pipe elements are not parallel to the global coordinate system $\bar{xy}$ for the entire pipe such as shown in Figure 5. Thus, the property matrices of most pipe elements must be transformed into the corresponding ones in terms of the global coordinate system before they are assembled to give the overall ones. For this reason, the transformation matrix $[\lambda]$ is introduced as follows (Wu, 2013)

$$
[\lambda] = \begin{bmatrix}
\cos \alpha & \sin \alpha & 0 & 0 & 0 \\
-\sin \alpha & \cos \alpha & 0 & 0 & 0 \\
0 & 0 & 1 & 0 & 0 \\
0 & 0 & 0 & \cos \alpha & \sin \alpha \\
0 & 0 & 0 & -\sin \alpha & \cos \alpha \\
0 & 0 & 0 & 0 & 1
\end{bmatrix}
$$

(8)

where

$$
\cos \alpha = \Delta x / \ell, \ \sin \alpha = \Delta y / \ell
$$

(9a,b)

$$
\Delta x = \bar{x}_2 - \bar{x}_1, \ \Delta y = \bar{y}_2 - \bar{y}_1, \ \ell = \sqrt{(\Delta x)^2 + (\Delta y)^2}
$$

(10a,b,c)

In the above equations, $\alpha$ is the angle between the positive local $x$-axis and the positive global $\bar{x}$-axis, while $(\bar{x}_1, \bar{y}_1)$ and $(\bar{x}_2, \bar{y}_2)$ are the global coordinates of the $1^{st}$ node $\odot$ and $2^{nd}$ node $\odot$ of the pipe element, respectively. It is evident that the symbol $\ell$ given by Equation (10c) denotes the length of the pipe element.

**Figure 5: The Node Displacements of a C-C Pipe Element with Respect to the Local Oxy Coordinate System, $\bar{u}_i$ (i = 1 – 6), and the Corresponding Ones with Respect to the Global $\bar{xy}$ Coordinate System, $\bar{\pi}_i$ (i = 1 – 6)**

Now, the element property matrices $[a]_{e}$ with respect to the local $oxy$ coordinate system given by Eqs. (1)-(6) can be transformed into the corresponding ones $[\pi]_{e}$ respect to the global $\bar{xy}$ coordinate system by using
\[ [\ddot{\mathbf{u}}] = [\lambda]^2 [\mathbf{a}][\lambda] \]  

(11)

where \([\mathbf{a}] = [m]_e, [c]_e, [k_e]_e, [k_f]_e\) or \([k_c]_e\), while \([\mathbf{a}] = [\overline{m}]_e, [\overline{c}]_e, [\overline{k_e}]_e, [\overline{k_f}]_e\) or \([\overline{k_c}]_e\).

**DETERMINATION OF UN-DAMPED NATURAL FREQUENCIES AND MODE SHAPES**

The equations of motion for the entire S-pipe take the form

\[ ([\overline{m}][\ddot{\mathbf{u}}] + [\overline{c}][\dot{\mathbf{u}}] + [\overline{k}][\mathbf{u}] = \{\overline{F}\} \]  

(12a)

where \([\overline{m}]\), \([\overline{c}]\) and \([\overline{k}]\) are the overall mass, damping and stiffness matrices of the entire pipe system, and \([\overline{F}]\) is the overall external loading vector. For free vibrations, one has \([\overline{F}] = 0\), thus, Equation (12a) becomes

\[ ([\overline{m}][\ddot{\mathbf{u}}] + [\overline{c}][\dot{\mathbf{u}}] + [\overline{k}][\mathbf{u}] = 0 \]  

(12b)

If the damping effect is neglected, then \([\overline{c}] = 0\), and Equation (12b) reduces to

\[ ([\overline{m}][\ddot{\mathbf{u}}] + [\overline{k}][\mathbf{u}] = 0 \]  

(12c)

Which is the standard equation of motion for an un-damped free vibration system and can be solved for the natural frequencies and mode shapes using the generalized Jacobi method (Bathe, 1982; Wu, 2013).

**DETERMINATION OF DAMPED NATURAL FREQUENCIES AND MODE SHAPES**

Equation (12b) is the equation of motion for a damped free-vibration system. Due to the existence of damping term, application of the existing computer codes (Garbow, 1977) requires that Equation (12b) must be transformed into the form below (Meirovitch, 1967; Wu et al., 2015)

\[ ([\overline{M}][\ddot{\mathbf{U}}] + [\overline{K}][\mathbf{U}] = 0 \]  

(13)

where

\[ [\overline{M}] = \begin{bmatrix} 0 & [\overline{m}] \\ [\overline{m}] & [\overline{c}] \end{bmatrix}, \quad [\overline{K}] = \begin{bmatrix} -[\overline{m}] & [0] \\ [0] & [\overline{k}] \end{bmatrix}, \quad [\mathbf{U}] = \begin{bmatrix} [\ddot{\mathbf{u}}] \\ [\overline{u}] \end{bmatrix}, \quad [\mathbf{U}] = \begin{bmatrix} [\ddot{\mathbf{u}}] \\ [\overline{u}] \end{bmatrix} \]  

(14a-d)

With each specified flowing velocity \(V\), the \(r\)th eigenvalues \(\omega_r\) and the associated \(r\)th eigenvectors \([\phi_r]\) of Equation (13) take the pairs of conjugate complex forms

\[ \omega_r = \omega_{r,R} \pm j\omega_{r,I}, \quad [\phi_r] = \begin{bmatrix} \omega_{r,R}\phi_{r,R} \\ \omega_{r,I}\phi_{r,I} \end{bmatrix} \]  

(15a,b)

In the above equations, the subscripts, \(R\) and \(I\), refer to “real” and “imaginary” parts of the complex numbers, respectively, and \(j = \sqrt{-1}\). Furthermore, the “imaginary” part of \(\omega_r\), \(\omega_{r,I}\), denotes the \(r\)th natural frequency and the corresponding “real” part of \([\phi_r]\), \([\phi_{r,R}]\), denotes the \(r\)th mode shape. Numerical results reveal that \([\phi_{r,R}] \neq [\phi_{r,I}]\), this may be due to \(\omega_{r,R} \neq \omega_{r,I}\).
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It is similar to the C-F pipe with free end at downstream (Chen and Fan, 1987; Aldraihem, 2007; Wu et al., 2015) that, for the F-C pipe studied in this paper, the following momentum transport at the upstream free end must be considered:

\[-m_j V^2 \bar{u}_i - m_j \dot{V} \bar{u}_i \mid_{x=0}.

**FORCED VIBRATION ANALYSIS BY FINITE ELEMENT METHOD**

In this section, the forced dynamic responses of the free-clamped (F-C) S-pipe subjected to the flow-induced impulsive forces at inlet and outlet and the centrifugal forces (CFs) at the 1st and 2nd curved pipe segments are studied.

The centrifugal force per unit pipe length induced by the water flowing through the curved pipe segments is given by

\[ F_c = \rho_w A_w V^2 / R \]  \hspace{1cm} (16)

where \( \rho_w \) is the mass density of water, \( A_w = A_j \) is the cross-section area of water column inside the pipe, \( V \) is average flow velocity and \( R \) is the radius of curvature of the curved pipe segment.

If the symbols \( n_{c1} \) and \( n_{c2} \) denote the total numbers of arc elements composing of the 1st and 2nd curved pipe segments, respectively, while \( \Delta \theta_1 \) and \( \Delta \theta_2 \) denote the subtended angles for each of the arc elements, then

\[ \Delta \theta_1 = \theta_1/n_{c1} = 40^\circ/16 = 2.5^\circ, \quad \Delta \theta_2 = \theta_2/n_{c2} = 40^\circ/20 = 2^\circ \]  \hspace{1cm} (17a,b)

Where \( \theta_1 = 40^\circ \) and \( \theta_2 = 40^\circ \) are the subtended angles for the first and second curved pipe segments, respectively, while \( n_{c1} = n_{c,2} = 16 \) and \( n_{c2} = n_{c,4} = 20 \) as one may see from Figure 3.

If the node numbering for the 1st node in the 1st curved pipe segment is denoted by \( n_{v,1,L} \), then the angle \( \theta_{1,i} \) between the centrifugal force \( F_{c,i} \) and the negative \( y \)-axis is determined by (cf. Figure 6)

\[ \theta_{1,i} = (n_{i} - n_{v,1,L}) \cdot \Delta \theta_1 \]  \hspace{1cm} (18)

In Figure 6, \( \vec{x}\vec{y} \) is the global coordinate system with origin \( \vec{x} \) at the inlet of the S-pipe.

Similarly to Equation (18), if the node numbering for the 1st node in the 2nd curved pipe segment is denoted by \( n_{v,2,L} \), then the angle \( \theta_{2,j} \) between the centrifugal force \( F_{c,2} \), and the positive \( y \)-axis is determined by

\[ \theta_{2,j} = \theta_2 - [(n_{j} - n_{v,2,L}) \cdot \Delta \theta_2] = 40^\circ - [(n_{j} - n_{v,2,L}) \cdot \Delta \theta_2] \]  \hspace{1cm} (19)
Impact Factor (JCC): 4.9875
NAAS Rating 3.04

Figure 6: The Centrifugal Force $F_{ci}$ at $i$th Node in the 1st Curved Pipe Segment with Angle $\theta_{1,i}$, with the $-\bar{y}$ Axis and its Components, $F_{ci,x}$ and $F_{ci,y}$, in $\bar{x}$ and $\bar{y}$ Directions, Respectively, and the Corresponding Ones at $i$th Node in the 2nd Curved Pipe Segment with Angle $\theta_{2,i}$ with the $+\bar{y}$ Axis, $F_{2i,x}$, $F_{2i,y}$, and $F_{2i,y}$. In Addition, $F_L = \rho_L A_L V_L^2$ and $F_R = \rho_R A_R V_R^2$ are the Impulsive Forces of the Flowing Fluid on the Left Inlet and Right Outlet of the Pipe, Respectively.

Based on the angle $\theta_{1,i}$, between each centrifugal force $F_{ci}$ and negative $\bar{y}$-axis given by Equation (18) for the 1st curved pipe segment and the corresponding one $\theta_{2,i}$ for the 2nd curved pipe segment, one may obtain the following centrifugal force components in $\bar{x}$-direction and those in $\bar{y}$-direction (cf. Figure 6)

$$F_{ci,x} = +F_{ci} \sin \theta_{1,i} \cdot F_{ci,y} = -F_{ci} \cos \theta_{1,i} \text{ (for } i = n_{1,L} \text{ to } (n_{1,L} + n_{1,1}) \text{ )}$$

$$F_{2i,x} = -F_{2i} \sin \theta_{2,i} \cdot F_{2i,y} = +F_{2i} \cos \theta_{2,i} \text{ (for } i = n_{2,L} \text{ to } (n_{2,L} + n_{2,2}) \text{ )}$$

Since the centrifugal force per unit length, $F^r_i$, is given by Equation (16), the distributed centrifugal node forces appearing in Eqs. (20a,b) and (21a,b) are determined by

$$F_{cix} = F^r_i \cdot R_1 \Delta \theta_1 = \rho_v A_v V^2 \cdot \Delta \theta_1 \text{ (for } i = (n_{1,L} + 1) \text{ to } (n_{1,L} + n_{1,1} - 1) \text{ )}$$

$$F_{c2x} = F^r_i \cdot R_2 \Delta \theta_2 = \rho_v A_v V^2 \cdot \Delta \theta_2 \text{ (for } i = (n_{2,L} + 1) \text{ to } (n_{2,L} + n_{2,2} - 1) \text{ )}$$

Eqs. (22a) and (23a) give the intermediate-node forces, and those at the two ends of the 1st and 2nd curved pipe segments are given by

$$F_{c1x} = \frac{1}{2} \rho_v A_v V^2 \cdot \Delta \theta_1 \text{ (for } i = n_{1,L} \text{ or } n_{1,L} + n_{1,1})$$

$$F_{c2x} = \frac{1}{2} \rho_v A_v V^2 \cdot \Delta \theta_2 \text{ (for } i = n_{2,L} \text{ or } n_{2,L} + n_{2,2})$$

It is noted that, in Eqs. (22a,b) and (23a,b), the units of $\Delta \theta_1$ and $\Delta \theta_2$ are “radian”.

RAYLEIGH DAMPING MATRIX

Since the damping matrix $[\mathbf{C}]$ appearing in Equation (12a) contains the effect of Coriolis force only, a Rayleigh damping matrix to include the overall damping effect of the entire S-pipe is introduced here, which is to take the form (Bathe, 1982; Wu, 2013)
\[ [\ddot{c}] = a [\bar{m}] + b [\bar{k}] \]  
(24a)

where \([\bar{m}]\) and \([\bar{k}]\) are the overall mass matrix and stiffness matrix, respectively, while \(a\) and \(b\) are constants determined by

\[
a = \frac{2\omega_j \omega_j (\xi_j \omega_j - \xi_i \omega_i)}{\omega_j^2 - \omega_i^2}, \quad b = \frac{2(\xi_j \omega_j - \xi_i \omega_i)}{\omega_j^2 - \omega_i^2} \quad (24b,c)
\]

In the above two expressions, \(\xi_i\) and \(\xi_j\) are the damping ratios corresponding to the two natural frequencies \(\omega_i\) and \(\omega_j\), respectively, for \(\omega_i < \omega_j\). In such a case, Equation (12a) becomes

\[
[\bar{m}][\ddot{u}] + ([\bar{c}] + [\ddot{c}])[\dot{u}] + [\bar{k}][u] = \{F\}
\]
(25)

Even if the effect of \([\bar{c}]\) due to Coriolis force is considered, from the subsequent studies, one may find that the forced vibration responses will become very large if one sets \([\ddot{c}] = 0\), because of neglecting the structural damping (i.e., \(\xi_i = \xi_j = 0\)).

**NUMERICAL RESULTS AND DISCUSSIONS**

In the finite element analysis, one of the tedious works is the preparation of input data. Thus, in this section, the determinations of global coordinates for the 67 nodes of the entire S-pipe and the other associated given data are introduced first. Next, the free and forced vibration-analysis results are followed.

As shown in Figure 3, the entire finite element model along the center line of the S-pipe is composed of five pipe segments including three straight ones (\(AB, DE\) and \(GH\)) and two curved ones (\(BCD\) and \(EFG\)), and the total numbers of pipe elements are \(n_{A} = 5\), 16, 16, 20 and 9, respectively. Where the lengths of elements in each pipe segment are equal to each other (but the length of each element in one pipe segment is different from that in the other pipe segment). Based on the above statements and the following information regarding the global coordinates of some key nodes, \(A(0, 0)\), \(B(5, 0)\), \(D(17, 856, 4, 679)\), \(E(27, 716, 12, 981)\), \(G(47, 20)\) and \(H(56, 20)\), one can obtain the global coordinates of all nodes by using computer.

The other associated given data are as follows: inner diameter of the double-shell pipe: \(d_i = 10\) m (or radius \(r_i = 5.0\) m); outer diameter of the double-shell pipe: \(d_o = 11.66\) m (or radius \(r_o = 5.83\) m); thickness of inner shell: \(t_i = 0.035\) m; thickness of outer shell: \(t_o = 0.032\) m; Young’s modulus: \(E = 200\) Gpa = \(2.0 \times 10^{11}\) N/m²; shear modulus: \(G = 79.3\) Gpa = \(0.793 \times 10^{11}\) N/m²; mass density of pipe shells: \(\rho_p = 7850\) kg/m³; mass density of water: \(\rho_w = 1000\) kg/m³; stiffness of soil: \(k_s = 203.6 \times 10^6\) N/m²; total pipe length: \(L = 61.795\) m; total pipe mass: \(m = 20.52289 \times 10^6\) kg (including mass of cement in ballast tanks); pipe mass per unit length: \(m' = m/L = 0.3321125 \times 10^6\) kg/m (Theoretical value: \(m' = \rho\ pA_{wall} = 17838.18\) kg/m = \(0.1783818 \times 10^3\) kg/m); pipe’s added mass per unit length: \(m'' = \rho\ p^2A_o = \rho\ p^2(\pi d_o^2/4) = 106779.273\) kg/m. The polar moments of inertia for the cross-sectional area of the outer and inner shells are \(I_{x, o} = I_{p, out} = \pi d_o^4/32 = [D_{o,2} - D_{o,1}^2]/4 = 39.51466\) m².
\( I_{xy} = I_{yz} = \frac{D_2^4 - D_1^4}{32} \) \( D_2 \). Thus, the moment of inertia of the entire double-shell pipe about \( y \)- or \( z \)- axis is \( I_y = I_z = \frac{1}{4}(I_{xx} + I_{yy}) = 33.64679 \, \text{m}^4 \).

In the foregoing expressions, the values of soil stiffness \( k_f \) and total pipe mass \( m \) are obtained from measurements and provided by the company building the S-pipe.

**FREE VIBRATION ANALYSIS**

In this subsection, the influence of mass density \( \rho_w = \rho_f \) and flowing velocity \( V \) of fluid on the natural frequencies and the associated mode shapes is studied.

**Influence of Mass Density of Fluid on Free Vibration Characteristics**

From Table 1 one sees that if the draft \( h = 30 \, \text{m} \) (at the outlet of the S-pipe) is selected to estimate the maximum flowing velocity of fluid in the pipe, then one has \( V = 24.25 \, \text{m/s} \). In such condition, the lowest five natural frequencies \( \omega_r (r/s) \) of the free-clamped (F-C) pipe are listed in Table 2 with the fluid mass densities \( \rho_w \) to be 1000 and 2000 \( \text{kg/m}^3 \), respectively. For the case of damping effect due to Coriolis force to be considered, the eigenvalues of the S-pipe are the complex numbers of the form \( \omega_r = \omega_{r,R} \pm j\omega_{r,I} \), thus, in addition to the imaginary parts \( \omega_{r,I} \) corresponding to the conventional un-damped natural frequencies \( \omega_r \), the real parts \( \omega_{r,R} \) are also listed in the parentheses of Table 2, because they influence the dynamic stability of a fluid-conveying pipe. From Table 2, one sees that:

- No matter the fluid mass density \( \rho_w = 1000 \) or 2000 \( \text{kg/m}^3 \), the damped natural frequency \( \omega_{r,I} \) is less than the corresponding un-damped one \( \omega_r \) (\( r = 1 - 5 \)), except the 2\(^{nd}\) and 5\(^{th}\) ones due to the negative damping effect of Coriolis force. However, the influence of Coriolis force is negligible. Since the real parts of all eigenvalues, \( \omega_{r,R} \), listed in the parentheses of Table 2 are negative, the dynamic instability due to Coriolis force will not occur for the present S-pipe.

- With \( \rho_w = 1000 \, \text{kg/m}^3 \), the corresponding lowest five mode shapes are shown in Figure 7(a) for the un-damped pipe and in Figure 7(b) for the damped pipe. Because the un-damped natural frequencies \( \omega_r (r = 1 - 5) \) are very close to the corresponding damped ones \( \omega_{r,I} \) as shown in Table 2, the un-damped mode shapes shown in Figure 7(a) are very close to the corresponding damped ones shown in Figure 7(b), except the 5\(^{th}\) mode shapes.
Table 2: The Lowest Five Un-Damped Natural Frequencies $\omega_1$, and Damped Ones $\omega_{1,j}$ of the S-Pipe Conveying Fluid with Velocities $V = 24.25$ m/s

<table>
<thead>
<tr>
<th>Conditions</th>
<th>Natural Frequencies, $\omega_1$ or $\omega_{1,j}$ (rad/s)</th>
<th>Damping effect (Coriolis force)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mass density of fluid (kg/m$^3$)</td>
<td>$\omega_1$, or $\omega_{1,j}$</td>
<td>$\omega_2$, or $\omega_{2,j}$</td>
</tr>
<tr>
<td>$\rho_w = 1000$</td>
<td>Neglected (un-damped)</td>
<td>18.7589</td>
</tr>
<tr>
<td></td>
<td>Considered (damped)</td>
<td>18.7527 (-0.0891)</td>
</tr>
<tr>
<td>$\rho_w = 2000$</td>
<td>Neglected (un-damped)</td>
<td>16.0995</td>
</tr>
<tr>
<td></td>
<td>Considered (damped)</td>
<td>16.0872 (-0.1311)</td>
</tr>
</tbody>
</table>

Figure 7: The Lowest Five Mode Shapes of the S-Pipe Conveying Fluid with Velocity $V = 24.25$ m/s and Mass Density $\rho_w = 1000$ kg/m$^3$ for the Cases of Coriolis Force (Damping) effect: (a) Neglected and (b) Considered

Influence of Flowing Velocity on Free Vibration Characteristics

The influence of flowing velocity $V$ of fluid on the property matrices of a pipe element is to appear in Equation (2) for the damping matrix due to Coriolis force and in Equation (7) for the centrifugal force associated with geometric stiffness matrix given by Equation (6). In Table 3, the 3rd row lists the lowest five natural frequencies of the pipe for the case of $V = 0$. From Table 3 one sees that, for the cases of $V = 10–30$ m/s, the influence of flowing velocity on the lowest five natural frequencies is small. For the “un-damped” natural frequencies $\omega_1$ of the pipe, the row before the final one of Table 3 indicates that $\omega_1$ and $\omega_3$ “increase” with the increase of flowing velocity $V$, and this trend is reverse for the $\omega_2$, $\omega_4$ and $\omega_5$. Furthermore, all the “damped” natural frequencies $\omega_{1,j}$ “decrease” with increasing the flowing velocity as shown in the final row of Table 3. Since the real parts of all eigenvalues, $\omega_{1,R}$, listed in the parentheses of Table 3, are negative, the dynamic instability due to Coriolis force will not occur for the present S-pipe. This conclusion agrees with...
that obtained from Table 2.

Table 3: The Lowest Five Un-Damped Natural Frequencies \( \omega_r \) and Damped Ones \( \omega_{r,i} \) of the S-Pipe Conveying Fluid with Mass Density \( \rho_w = 1000 \text{ kg/m}^3 \) and Velocities \( V = 0, 10, 15, 20, 25 \) and 30 m/s, Respectively

<table>
<thead>
<tr>
<th>Conditions</th>
<th>Natural Frequencies, ( \omega_r ) or ( \omega_{r,i} ) (rad/s)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Flowing Velocity, ( V ) (m/s)</td>
<td>Damping effect (Coriolis force)</td>
</tr>
<tr>
<td>0</td>
<td>Neglected (un-damped)</td>
</tr>
<tr>
<td>10</td>
<td>Considered (damped)</td>
</tr>
<tr>
<td>20</td>
<td>Considered (damped)</td>
</tr>
<tr>
<td>30</td>
<td>Considered (damped)</td>
</tr>
<tr>
<td>Trend of frequencies</td>
<td>Un-damped Increasing Decaying Increasing Decreasing Decreasing</td>
</tr>
</tbody>
</table>

FORCED VIBRATION ANALYSIS

The objective of this section is to study the dynamic responses of the S-pipe due to actions of the impulsive forces and the centrifugal forces induced by the flowing fluid with velocity

\[
V(t) = V_0 (1 + \delta \cos \omega_r t) \tag{26}
\]

For convenience, the following numerical values are used:

\[
V_0 = 24.25 \text{ m/s (or } h = 30 \text{ m)} , \quad \rho_w = 1500 \text{ kg/m}^3 , \quad \omega_r = 1 - 36 \text{ rad/s} , \quad \xi_1 = \xi_2 = 0.05 \tag{27}
\]

Time Histories of Displacements at Nodes 1 and 38

Based on the conditions given by Equation (27) with pulsating frequency \( \omega_r = 10 \text{ rad/s} \) and pulsating parameter \( \delta = 0 \), the time histories for the displacements of nodes 1 and 38 in the horizontal (\( \bar{x} \)) direction, \( u_1(t) \) and \( u_{112}(t) \), are shown in Figure 8(a), and the corresponding ones in the vertical (\( \bar{y} \)) direction, \( u_2(t) \) and \( u_{113}(t) \), are shown in Figure 8(b). It is seen that all curves become the horizontal lines after time \( t \geq 7 \text{ seconds} \) due to the effect of pipe’s structural damping. Since the pulsating parameter is \( \delta = 0 \) for Figure 8, the ordinates corresponding to the last horizontal lines are,
respectively, called the “quasi-static deflections” (\(\Delta x\) or \(\Delta y\)) of the S-pipe at the specified nodes in the horizontal (\(\bar{X}\)) or vertical (\(\bar{Y}\)) directions due to actions of the “constant” fluid impulses at the inlet and outlet as well as the centrifugal forces at the 1st and 2nd curved pipe segments. From Figure 8(a), one sees that \(\Delta_{1x} \approx 0.021\) m and \(\Delta_{38x} \approx 0.0049\) m. Similarly, from Figure 8(b), one sees that \(\Delta_{1y} \approx -0.021\) m and \(\Delta_{38y} \approx 0.0035\) m. It is noted that all “quasi-static deflections” are positive (either rightward or upward) except \(\Delta_{1y}\) (downward).

If all the foregoing conditions are kept unchanged except that the pulsating parameter of flowing velocity is replaced by \(\delta = 0.2\) (instead of \(\delta = 0\)), then the time histories of displacements at nodes 1 and 38 are shown in Figure 9(a) for the horizontal (\(\bar{X}\)) displacements, \(u_1(t)\) and \(u_{112}(t)\), and in Figure 9(b) for the vertical (\(\bar{Y}\)) displacements, \(u_2(t)\) and \(u_{113}(t)\). It is under our expectation that all curves become those of harmonic motions with respect to their “quasi-static” equilibrium positions (q-SEPs) after time \(t \geq 7\) seconds, because the influences of all initial velocities have been damped out.

Figure 8: For the Case of \(\delta = 0\), \(\omega = 10\) Rad/s, \(\rho = 1500\) kg/m\(^3\), \(V_0 = 24.25\) m/s and \(\xi_1 = \xi_2 = 0.05\), the time Histories of Displacements at Nodes 1 and 38: (a) Horizontal (\(\bar{X}\)) Displacements, \(u_1(t)\) and \(u_{112}(t)\); (b) Vertical (\(\bar{Y}\)) Displacements, \(u_2(t)\) and \(u_{113}(t)\)
For the conditions given by Equation (27), the lowest five un-damped natural frequencies of the S-pipe are: 17.2773, 19.5136, 28.8829, 57.3030 and 66.2902 rad/s, respectively. Based on the centrifugal forces (CFs) on the 1st and 2nd curved pipe segments along with the impulsive forces on the inlet and outlet of the pipe shown in Figure 6, the frequency-response amplitude curves due to $\delta = 0.2$ and 0.4 are as follows:

For the case of $\delta = 0.2$, the frequency-response amplitude curves for nodes 1 and 38 are shown in Figure 10(a) for the horizontal ($\bar{x}$) displacement amplitudes, $|u_1(t)|_{\text{max}}$ and $|u_{112}(t)|_{\text{max}}$, and in Figure 10(b) for the vertical ($\bar{y}$) displacement amplitudes, $|u_2(t)|_{\text{max}}$ and $|u_{113}(t)|_{\text{max}}$. Among the four curves, the solid ones are for node 1 and the dashed ones are for node 38. From Figure 10(a) one sees that each curve has a peak when the pulsating frequency $\omega_1$ approaches the first un-damped natural frequency $\omega_1$ of the pipe (i.e., $\omega_1 \approx 17.2773$ rad/s), this is because the both horizontal displacements are in the rightward ($+\bar{x}$) direction. However, from Figure 10(b) one sees that the solid curve for the vertical downward ($-\bar{y}$) displacements of node 1 has a peak at $\omega_2 \approx \omega_1 = 17.2773$ rad/s, and the dashed curve for the vertical upward ($+\bar{y}$) displacements of node 38 has a peak at $\omega_2 \approx \omega_1 = 19.5136$ rad/s. It is seen that either horizontal or vertical displacement amplitudes, those of node 1 at the left free end of the pipe are much greater than the corresponding ones at the intermediate node 38. In addition, the maximum vertical displacement amplitude at node 1,
$| -u_1(t)|_{\text{max}} \approx 0.125 \text{m}$ shown in Figure 10(b) is much greater than the corresponding quasi-static deflection $| -\Delta_{12}| \approx 0.021 \text{m}$ shown in Figure 8(b) or 9(b). This result indicates that the S-pipe is safe under constant (quasi-static) loads and may be questionable under pulsating (dynamic) loads, particularly because it is deeply under the free water surface (with $h_{\text{max}} = 35 \text{ m}$).

**Figure 10:** For the Case of $\delta = 0.2$ and the Conditions given by Equation (27), the Frequency-Response Amplitude Curves for Nodes 1 and 38: (a) Horizontal ($\overline{X}$) Displacement Amplitudes, $|u_1(t)|_{\text{max}}$ and $|u_{112}(t)|_{\text{max}}$; (b) Vertical ($\overline{Y}$) Displacement Amplitudes, $| -u_2(t)|_{\text{max}}$ and $|u_{113}(t)|_{\text{max}}$

If all conditions for Figure 10 are kept unchanged except that $\delta = 0.4$, then, the frequency-response amplitude curves for nodes 1 and 38 are shown in Figure 11(a) and (b), respectively. It is seen that, either the horizontal or vertical displacement amplitudes, the peak values shown in Figure 11(a) and (b) for the case of $\delta = 0.4$ are much greater than the corresponding ones shown in Figure 10(a) and (b) for the case of $\delta = 0.2$. This result is under our expectation, because $|V(t)|_{\text{max}} = 1.2V_0$ if $\delta = 0.2$, and $|V(t)|_{\text{max}} = 1.4V_0$ if $\delta = 0.4$, so that the ratio between the exciting force amplitudes for the last two cases is $|F(t)|_{\delta=0.4}/|F(t)|_{\delta=0.2} = (1.4V_0)^2/(1.2V_0)^2 \approx 1.361$. From Figure 11(b), one obtains the maximum vertical displacement amplitude at node 1, $| -u_2(t)|_{\text{max}} \approx 0.227 \text{m}$, which is 1.816 times the corresponding one $| -u_2(t)|_{\text{max}} \approx 0.125 \text{m}$ obtained from Figure 10(b) for the case of $\delta = 2.0$ and 10.81 times the quasi-static deflection $| -\Delta_{12}| \approx 0.021 \text{m}$ obtained from Figure 8(b) for the case of $\delta = 0$. It is noted that $0.227 \text{m}/0.125 \text{m} = 1.816$, this is greater than the exciting force amplitude ratio 1.361, due to the dynamic magnification effect. Such a nonlinear raise of dynamic response amplitudes of the S-pipe with the increase of pulsating parameter $\delta$ may be the negative information for the safety of the S-pipe after the operation.
CONCLUSIONS

Based on the foregoing numerical results and discussions, one obtains the following conclusions:

• For the actual S-pipe studied in this paper, the influence of water-surface elevation of the reservoir on its lowest five natural frequencies is negligible, however, the variation of fluid mass density (due to sediments) influences its free vibration characteristics to some degree.

• Because the damping effect due to Coriolis force is negligible, either the “quasi-static” or the “dynamic” deflections of the actual S-pipe at its free end (inlet) will become very large if the structural damping effect in not considered. In such a case, introducing the Rayleigh damping matrix in terms of damping ratios ($\xi_1$ and $\xi_2$) of the entire vibrating system will be much better than using the inner and outer damping coefficients ($c_i$ and $c_o$) of each components, because the determination of damping ratios is easier and more practical than that of damping coefficients.

• For the S-pipe studied in this paper, the flowing velocity of fluid will induce a pair of impulsive forces at its inlet and outlet, as well as a pair of centrifugal forces at its first and second curved pipe segments. The pair of impulsive forces are equal to each other in magnitudes, opposite in directions and not collinear, thus, they produce an unbalanced couple, and so do for the pair of centrifugal forces. For the fluid flow with “constant” velocity $V_0$, the quasi-static deflection at inlet (located at free end) is negligible, however, for the fluid flow with “pulsating” velocity $V(t) = V_0(1 + \delta \cos \omega t)$ and the pulsating frequency $\omega_p$ approaching the first natural frequency $\omega_1$ of the pipe, the maximum deflection at inlet may be more than ten times that of the quasi-static deflection (dependent on the magnitude of pulsating parameter $\delta$). In such situation, the safety of the S-pipe may be questionable.
Particular attention should be paid to the liquefaction of soil contacting with the S-pipe, because this may be the main reason leading to collapse of the S-pipe. It is evident that the general straight pipes do not have the foregoing problems.

- For the S-pipe studied in this paper, attentions must be paid to the operations of the gate at the pipe outlet. It is possible that one cannot open or close the gate due to its unusual size and the huge hydraulic pressure or impulsive force on it.

- From the view point of safety, efficiency or practicality, the “siphon” shown in Figure A1 (in Appendix of this paper) should be better than the actual S-pipe shown in Figure 1.

ACKNOWLEDGEMENTS

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Dynamic Analysis of the S-Type Fluid-Conveying Steel Pipe for Tsen-Weng Reservoir Using Finite Element Method

Englewood Cliffs, N.J.


APPENDIX: CLEANING DEPOSIT OF TSEN-WENG RESERVOIR BY USING SIPHON

For leaning the deposit of Tsen-Weng reservoir, instead of the conventional sluice gate near free water surface, the siphon with its inlet near reservoir bottom (Figure A1) will be safer, more efficient and more practical.

Figure A1: The Original Conception for Cleaning Deposit of Tsen-Weng Reservoir by Using Siphon Theory (C.V. = Control Valve)