International Journal of Human Resources Management (IJHRM)
ISSN(P): 2319-4936; ISSN(E): 2319-4944
Vol. 5, Issue 6, Oct - Nov 2016; 1-12


International Academy of Science, Engineering and Technology
Connecting Researchers; Nurturing Innovations

# FUZZY DIVISOR CORDIAL GRAPH

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## ABSTRACT

In this paper we introduce a new concept called fuzzy divisor cordial labeling. It is a conversion of crisp graph into fuzzy graph under the new condition namely fuzzy divisor cordial labeling. In divisor cordial labeling it is not possible to label all the crisp graphs due to the condition of its definition. Suppose if we consider a graph of size 5, it will be possible to label all the vertices as in the combination of vertex set {1,2,3,4,5}. So for n vertices, we need to label all the vertices as a combination of all the vertices without repetition, without neglecting any vertex among them. Here discussion about the edge labeling is trivial. So it is clear that all the crisp graphs can't be divisor cordial graphs. However in fuzzy divisor cordial graph for any vertices we can label any fuzzy membership value from [0,1]. Since the interval consists of infinite number of terms, there are infinite number of chances for labeling a vertex in fuzzy divisor cordial labeling.

**KEYWORDS:** Fuzzy Divisor Cordial Labeling

## INTRODUCTION

The existence of graphs for which a special set of integer values are assigned to its nodes or edges or both according to some given criteria has been investigated since the middle of the last century. Graph labeling have often been motivated by practical considerations such as chemical isomers, but they are also of interest in their own right due to their abstract mathematical properties arising from various structural considerations of the underlying graphs. The qualitative labelings of graph elements have inspired research in diverse field of human enquiry such as conflict resolution in social psychology, electrical circuit theory and energy crisis. Quantitative labelings of the graphs have led to quite intricate fields of applications such as coding theory problems, including the design of good radar location codes, synch-set codes, missile guidance codes and convolution codes with optimal auto correlation properties. Labeled graph often been applied in determining the ambiguities in X-ray crystallographic analysis, to design communication networks, in determining optimal circuit layouts and radio astronomy etc.

Interest in graph labeling problems began in the mid 1960's. Most graph labeling methods trace their origin to one introduced by Rosa in 1967, or one given by Graham and Sloane in 1980. The concept of cordial labeling was introduced by Cahit.

The definition of fuzzy graph was first introduced by Kaufmann in the year 1973, based on L.A. Zadeh's fuzzy relations, introduced in the year 1971. Then a mathematician Azriel Rosenfeld developed the theory of fuzzy graph who considered fuzzy relations on fuzzy sets and in 1975. R.T. Yeh and S.Y. Bang have also introduced various connectedness concepts in fuzzy graph in the same year. Yeh and Bang's approach for the study of fuzzy graphs were motivated by its applicability to pattern classification and clustering analysis. They worked more with the fuzzy matrix of a fuzzy graph, introduced concepts like vertex connectivity $\Omega(G)$ , edge connectivity $\kappa(G)$ and established the fuzzy analogue of Whiteney's theorem. They also proved that for any three real numbers a, b, c such that $0 < a \leq b \leq c$, there exists a fuzzy graph G with $\Omega(G) = a$, $\Lambda(G) = b$ and $\delta(G) = c$.

Fuzzy graphs have been witnessing a tremendous growth and finding applications in many branches of engineering and technology so far. Rosenfeld has obtained several concepts like bridges, paths, cycles, and trees and established some of their properties. Fuzzy end nodes and cut nodes were studied by K.R.Bhutani. Bhattacharya has established some connectivity concepts regarding fuzzy cut nodes and fuzzy bridges titled "Some remarks on fuzzy graphs".

In this paper we have introduced a new concept namely fuzzy divisor cordial graph as discussed above. It's just an application of labeling fuzzy numbers for the graphs under some new conditions. Fuzzy graph is the generalization of the crisp graph. So it is necessary to know some basic definitions and concepts of crisp graph based on divisor cordial graphs.

**Def:1.1**

The assignment of values subject to certain conditions to the vertices of a graph is known as graph labeling.

**Def: 1.2**

Let G = (V,E) be a graph. A mapping $f : V(G) \rightarrow \{0,1\}$ is called binary vertex labeling of G and f(v) is called the label of the vertex v of G under f.

For an edge e = uv, the induced edge labeling $f^* : E(G) \rightarrow \{0,1\}$ is given by $f^*(e) = |f(u)-f(v)| \leq 1$. Let $v_f(0)$ and $v_f(1)$ be the number of vertices of graph G having labels 0 and 1respectively under f and let $e_f(0)$ and $e_f(1)$ be the number of edges having labels 0 and 1 respectively under f*.

**Def: 1.3**

A binary vertex labeling of a graph G is called a cordial labeling if $|v_f(0)-v_f(1)| \leq 1$and $|e_f(0)-e_f(1)| \leq 1$. A graph G is cordial if it admits cordial labeling.

**Def: 1.4**

Let G = (V,E) be a simple graph and $f : V \rightarrow \{1,2,\ldots|V|\}$ be a bijection. For each edge uv, assign the label 1 if either f(u) divides f(v) or f(v) divides f(u) otherwise label 0. Then f is called a divisor cordial labeling of a graph G if $|e_f(0)-e_f(d)| \leq 1$.

**Def: 1.5**

A fuzzy graph $G = (\sigma,\mu)$ is a pair of functions $\sigma : V \rightarrow [0,1]$ and $\mu : V \times V \rightarrow [0,1]$, where for all $u,v \in V$, we have $\mu(u,v) \leq \sigma(u) \wedge \sigma(v)$.

**Def: 1.6**

A labeling of a graph is an assignment of values to the vertices and edges of a graph.

## MAIN RESULTS

**Def: 1.7**

A graph $G = (\sigma,\mu)$ is said to be a fuzzy labeling graph if

$\sigma : V \rightarrow [0,1]$ and $\mu : V \times V \rightarrow [0,1]$ is bijective such that the membership value of edges and vertices are distinct and $\mu(u,v) \leq \sigma(u) \wedge \sigma(v)$ for all u, v $\in$ V.

**Def: 1.8**

Let G = $(\sigma,\mu)$ be a simple graph and $\sigma : V \rightarrow [0,1]$ be a simple bijection. For each edge uv, assign the label d if either $\sigma(u) \mid \sigma(v)$ or $\sigma(v) \mid \sigma(v)$ and the label 0 if $\sigma(u) \dagger \sigma(v)$. $\sigma$ is called a fuzzy divisor cordial labeling if $\left| e_\mu(0)-e_\mu(d) \right| \leq 1$, where d is a very small positive quantity, which is closure to 0 and d $\in$ (0,1).

In other words a graph with a fuzzy divisor cordial labeling is called a fuzzy divisor cordial labeling graph.

**Def: 1.9**

The graph $p_{n-1}(1,2,3,\ldots n)$ is a graph obtained from a path of vertices $v_1,v_2,\ldots,v_n$ having path length n-1 by joining i pendent vertices of each of $i^{th}$ vertex. The pendent vertices are labeled as $u_{i,1}$ ; $u_{i,2}$ ;…; $u_{i,n}$ for $1 \leq i \leq n$.

**Def: 1.10**

The graph $p_{n-1}(k,k,\ldots k)$ is a graph obtained from a path vertices $u_1,u_2,\ldots u_n$ having path length n-1 by joining k pendent vertex at each path vertex $v_i$.

**Theorem: 1.11**

A complete n-nary tree is a fuzzy divisor cordial graph.

**Proof**

Let $v_0$ be the root of the tree, which is a zero level vertex.

Fix $\sigma(v_0)$ by any fuzzy number other than o and $1/10^i$, for all i $\in$ W, where W ia set of all whole numbers.

Let $\sigma(v_0)$ = x.

In level one, n vertices are there.

Suppose n is odd, then label $\frac{n-1}{2}$ or $\frac{n+1}{2}$ vertices by $x/10^i$, the remaining n + 1 vertices should have been labeled as the non-divisible membership value of x.

Therefore we have $e_\mu(0) = \frac{n+1}{2}$, $e_\mu(d) = \frac{n-1}{2}$.

Hence $\left| e_\mu(0)-e_\mu(d) \right| = \left| \frac{n+1}{2} - \frac{n-1}{2} \right| = 1$.

Therefore $\left| e_\mu(0)-e_\mu(d) \right| \leq 1$.

If $\frac{n+1}{2}$ vertices labeled $x/10^i$, remaining $\frac{n-1}{2}$ vertices should have been labeled as the membership values , which are not divisible by x.

So $e_\mu(0) = \frac{n-1}{2}$, $e_\mu(d) = \frac{n+1}{2}$.

Hence $\left| e_\mu(0)-e_\mu(d) \right| = \left| \frac{n-1}{2} - \frac{n+1}{2} \right| = \left| -1 \right|$.

Therefore $\left| e_\mu(0)-e_\mu(d) \right| \leq 1$ holds.

Note that if level increases then i will increase.

In level two, $n^2$ vertices are there, which are odd number of vertices.

Label $x/10^i$ for $(n^2+1)/2$ if already labeled $\frac{n-1}{2}$ in level one.

Therefore remaining $(n^2-1)/2$ vertices should be labeled by the membership value which can't be divided by $x/10^i$, without repeating repeating any $\sigma(v_j)$ is a membership value of any vertex.

So $e_\mu(0) = (n^2-1)/2$ and $e_\mu(d) = (n^2+1)/2$.

Therefore we have $|e_\mu(0)-e_\mu(d)| \leq |(n^2-1)/2 - (n^2+1)/2| = |-1|$.

Hence $|e_\mu(0)-e_\mu(d)| \leq 1$ holds.

Label x/10i for $(n^2-1)/2$ if already labeled $\frac{n+1}{2}$ in level one, remaining $(n^2+1)/2$ vertices should be labeled by the membership value which is not divisible by $x/10^i$, without repeating any $\sigma(v_j)$.

So that $e_\mu(0) = (n^2+1)/2$ and $e_\mu(d) = (n^2-1)/2$.

Therefore we have $|e_\mu(0)-e_\mu(d)| \leq |(n^2+1)/2 - (n^2-1)/2| = |1|$.

Hence $|e_\mu(0)-e_\mu(d)| \leq 1$ holds.

For level n, $n^n$ vertices are there.

Label $x/10^i$ for $(n^n-1)/2$ if we have labeled fort $(n^{n-1} + 1)/2$ in level n-1.

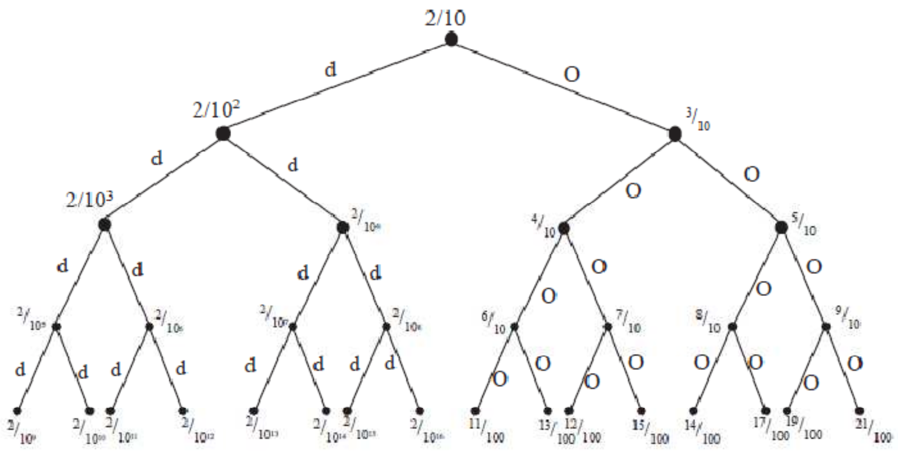
Label $x/10^i$ for $(n^n+1)/2$, if we have labeled for $(n^{n-1}-1)/2$ in level n-1.

For both, remaining vertices should be labeled as a membership value, which can't be divided by $x/10^i$ without repeating any $\sigma(v_j)$.

Therefore $|e_\mu(0)-e_\mu(d)| \leq 1$ holds

**Illustration: 1.12**

Consider a complete binary tree upto level three.




Here $e_\mu(0) = 15$, $e_\mu(d) = 15$

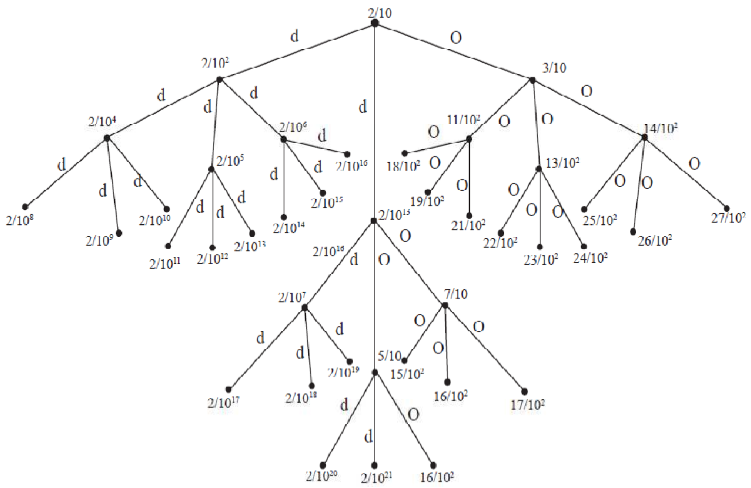
$\left| e_\mu(0) - e_\mu(d) \right| = \left| 15 - 15 \right| = 0 \le 1$

Hence $\left| e_\mu(0) - e_\mu(d) \right| \le 1$.

Therefore every complete binary tree is a fuzzy divisor cordial graph.

**Illustration: 1.13**

Consider a complete trinary tree upto level two.



Here $e_\mu(0) = 19$, $e_\mu(d) = 20$

$\left| e_\mu(0) - e_\mu(d) \right| = \left| 19 - 20 \right| = \left| -1 \right| \le 1$

Hence $\left| e_\mu(0) - e_\mu(d) \right| \le 1$.

Therefore every complete trinary tree is a fuzzy divisor cordial graph.

From *illustration 1.12 and illustration 1.13,* we can conclude that every n-nary tree is a fuzzy divisor cordial graph.

**Theorem: 1.14**

Every complete graph $k_n$ is a fuzzy divisor cordial graph.

**Proof**

For complete graphs $k_1$ and $k_2$ the proof is trivial,

i.e $\left| e_\mu(0)-e_\mu(d) \right| \le 1$ holds for the graphs $k_1$ and $k_2$. So we need to prove this result for $n \ge 3$.

In a complete graph $k_n$ $n \ge 3$, nth vertex say $v_n$ has n-1 contributions.

Vertex $v_{n-1}$ has n-2 contributions, proceeding in this manner $v_2$ has 1 contribution and $v_1$ has no contribution.

Let m be the total number of contributions in graph $k_n$, i.e $m = \frac{n(n-1)}{2}$.

**Part A**

Suppose the number of contributions of $v_n$ is greater than or equal to $\frac{m}{2}$ if m is even and the number of contributions of $v_n$ is greater than or equal to $\frac{m-1}{2}$ if m is odd.

Then fix $\sigma(v_n) = 1$.

Hence the remaining vertices $v_1, v_2, \ldots, v_{n-1}$ must be labeled by any fuzzy number which can't be divided by one another without repeating any vertex $\sigma(v_j)$

Therefore $|e_\mu(0)-e_\mu(d)| \leq 1$ holds.

**Part B**

Assume that the number of contribution of $v_n$ is less than $\frac{m}{2}$ if m is even and less than $\frac{m+1}{2}$ or $\frac{m-1}{2}$ if m is odd.

We know that half of the contribution must be connected with divisible labels to satisfy the condition of fuzzy divisor cordiality.

**Case (i)**

Suppose sum of contribution of $v_n$ and contribution of some other vertex $v_k$ is not less than half of the total contribution for both odd and even.

Then fix $\sigma(v_j) = 1$.

Next, label the necessary vertices as divisor of some other vertices, not all however to satisfy the condition of fuzzy divisor cordiality. Remaining vertices must be labeled by any fuzzy number which can't be divisible by one another.

Therefore $|e_\mu(0)-e_\mu(d)| \leq 1$ holds.

**Case (ii)**

Suppose sum of contributions of vertex $v_n$ and sum of some other vertices less than half of the total contribution for both odd and even.

Fix the membership value of those vertices as $1/10^i$ for all $i \in W$, where W is a set of all whole numbers.

Label the necessary vertices as a divisor of some other vertices except the above to satisfy the fuzzy divisor cordiality if necessary. The remaining vertices must be labeled by any fuzzy number which can't be divisible by one another without repeating any vertex $\sigma(v_j)$.

Hence $|e_\mu(0)-e_\mu(d)| \leq 1$ holds.

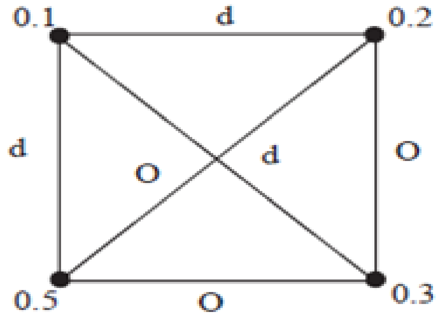



**Illustration: 1.15**

Consider a complete graph $k_4$.

Here $e_\mu(0) = 3$, $e_\mu(d) = 3$

$| \, e_\mu(0) - e_\mu(d) \, | = |3 - 3| = |0| \leq 1$

Hence $|e_\mu(0) - e_\mu(d)| \leq 1$.

Therefore complete complete graph $k_4$ is a fuzzy divisor cordial graph.

**Note**

In a normal crisp graph complete graph $k_4$ is not a divisor cordial graph. Also $k_7$ is not a divisor cordial graph. However fuzzy graphs $k_4$ and $k_7$ are fuzzy divisor cordial graph.

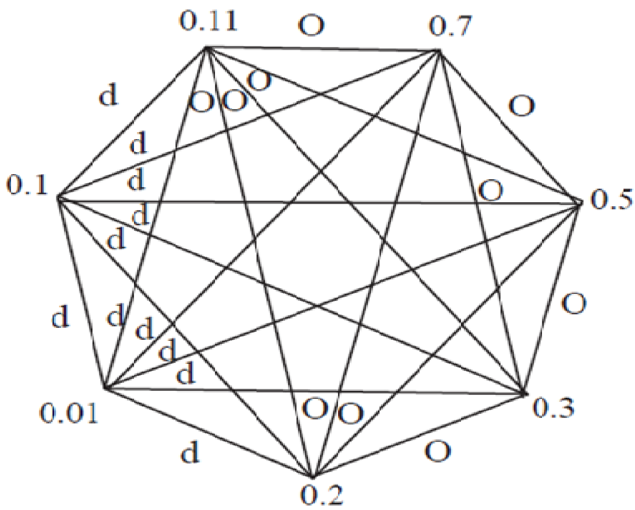
**Illustration: 1.16**

Consider a complete graph $k_7$.

Here $e_\mu(0) = 19$, $e_\mu(d) = 20$

$| \, e_\mu(0) - e_\mu(d) \, | = |10 - 11| = |-1| \leq 1$

Hence $|e_\mu(0) - e_\mu(d)| \leq 1$.




Therefore every complete complete graph $k_7$ is a fuzzy divisor cordial graph.

*From illustration 1.15 and illustration 1.16*, we can conclude that every complete graph $k_n$ is a fuzzy divisor cordial graph.

**Theorem: 1.17**

Every path $p_n$ is a fuzzy divisor cordial graph.

**Proof**

Let $v_0,v_1,v_2,\dots,v_n$ be the vertices of path $p_n$.

Fix $\sigma(v_0) = x$, where $x\in(0,1]$.

**Case (i)**

If n is even, there exists odd number of vertices and even number of edges.

Label the fuzzy number $x/10^i$ for all i upto $\frac{n+1}{2}$ or $\frac{n-1}{2}$ vertices.

If $x/10^i$ is labeled upto $\frac{n-1}{2}$ vertices, then the remaining $\frac{n+1}{2}$ vertices must be labeled by any fuzzy number, which can't be divisible by it's neighbours without repeating any vertex $\sigma(v_j)$ for all j.

This is only possible when $v_{(n-1)/2} = 1/10^i$ for all i.

Therefore $e_\mu(o) = \frac{n}{2}$ and $e_\mu(d) = \frac{n}{2}$

Therefore $|e_\mu(0)\text{-}e_\mu(d)| \le |\frac{n}{2}\text{-}\frac{n}{2}| = 0 \le 1$

Hence $|e_\mu(0)\text{-}e_\mu(d)| \le 1$ holds.

If $x/10^i$ is labeled upto $\frac{n+1}{2}$ then the remaining $\frac{n-1}{2}$ vertices must be labeled by any fuzzy number, which can't be divided by it's neighbours, without repeating any $\sigma(v_j)$ for all j.

This is possible only when $v_{(n+1)/2} \neq 1/10^i$ for all i.

Therefore by above , we have $|e_\mu(0)\text{-}e_\mu(d)| \le 1$

**Case (ii)**

If n is odd, there exist even number of vertices and odd number of edges.

Fix $\sigma(v_0) = x \in (0,1]$.

Label the fuzzy number $x/10^i$ for all i upto $\frac{n}{2}$ vertices.

Then the remaining $\frac{n}{2}$ vertices must be labeled by any fuzzy number which can't be divided by it's neighbours without repeating the membership value of vertex $v_j$ for all j.

If $v_{n/2} = 1/10^i$, then $v_{n/2}$ will divide succeeding vertx.

Therefore $e_\mu(o) = \frac{n}{2}$ -1and $e_\mu(d) = \frac{n}{2}$.

Hence $|e_\mu(0)-e_\mu(d)| \leq |\frac{n}{2}-1-\frac{n}{2}| = |-1|$

Therefore $|e_\mu(0)-e_\mu(d)| \leq 1$ holds.

If $v_{n/2} \neq 1/10i$, then $v_{n/2}$ will not divide the succeeding vertex.

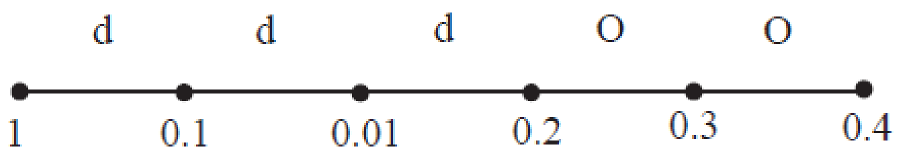
Therefore we have $e_\mu(o) = \frac{n}{2}$ and $e_\mu(d) = \frac{n}{2}-1$.

Hence $|e_\mu(0)-e_\mu(d)| \leq |\frac{n}{2}-(\frac{n}{2}-1)| = |1|$

Therefore $|e_\mu(0)-e_\mu(d)| \leq 1$ holds.

**Illustration: 1.18**

Consider the path p5.




Here $e_\mu(0) = 2$, $e_\mu(d) = 3$

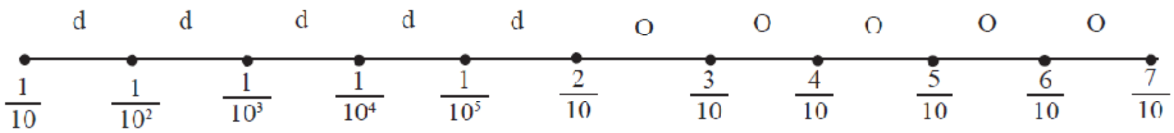
$|e_\mu(0) - e_\mu(d)| = |2 - 3| = |-1| \leq 1$

Hence $|e_\mu(0) - e_\mu(d)| \leq 1$.

Therefore every path $p_5$ is a fuzzy divisor cordial graph.

**Illustration: 1.19**

Consider a path $p_{10}$.




Here $e_\mu(0) = 5$, $e_\mu(d) = 5$

$|e_\mu(0) - e_\mu(d)| = |5 - 5| = |0| \leq 1$

Hence $|e_\mu(0) - e_\mu(d)| \leq 1$.

Therefore every path $p_{10}$ is a fuzzy divisor cordial graph.

Now from the illustration 5 and 6 we can conclude that every path $p_n$ is a fuzzy divisor cordial graph.

**Theorem: 1.20**

Every cycle $C_n$ is a fuzzy divisor cordial graph.

**Proof**

We know that cycle $C_n$ has n number of vertices and n number of edges.

Let $v_0, v_1, v_2, \ldots v_n$ be the vertices of cycle $C_n$.

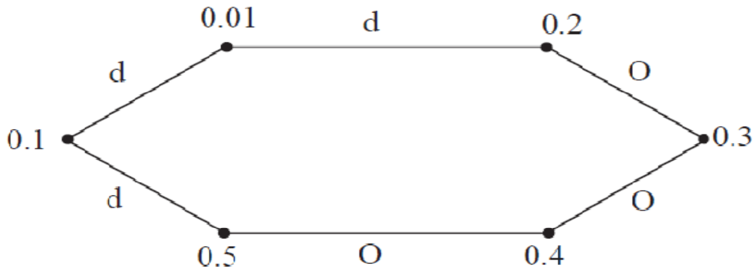
If n is odd, there exist odd number of edges.

Also if n is even, there exist even number of edges.

Therefore the proof is similar to theorem 3.

**Illustration: 1.21**

Consider cycle graphs $C_6$ and $C_7$.

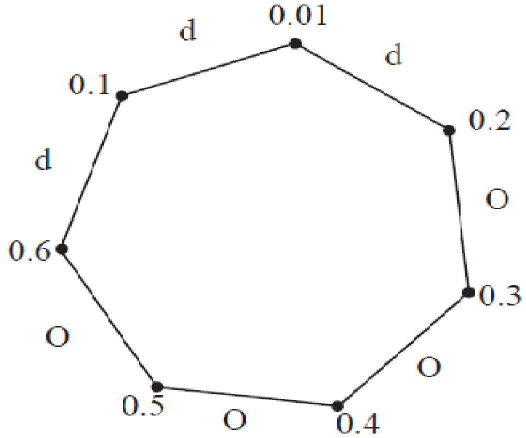



**Cycle $C_6$**

Here $e_\mu(0) = 3$, $e_\mu(d) = 3$

$| e_\mu(0) - e_\mu(d) | = | 3 - 3 | = | 0 | \leq 1$

Hence $| e_\mu(0) - e_\mu(d) | \leq 1$.




**Cycle $c_7$**

Here $e_\mu(0) = 4$ , $e_\mu(d) = 3$

$| e_\mu(0) - e_\mu(d) | = | 4 - 3 | = | 0 | \leq 1$

Hence $| e_\mu(0) - e_\mu(d) | \leq 1$.

Therefore every cycle $C_n$ is a fuzzy divisor cordial graph

**Theorem: 1.22**

Every wheel graph $W_n$ is a fuzzy divisor cordial graph.

**Proof**

In a graph , if there are n number of vertices, there will be 2(n-1) edges.

Therefore the wheel $W_n$ graph contains 2(n-1) edges.

Let $v_0, v_1, v_2, \ldots v_n$ be the vertices of a wheel graph $W_n$.

Fix $v_0$ at the centre.

Fix $\sigma(v_0) = 1/x^i$ for all $i \in W$, where W is a set of all whole numbers.

Therefore we can divide all the remaining vertices. Otherwise fix 0 for $\sigma(v_0)$.

If $1/10^i$ labels at the centre, then 0 must not be labeled as any vertex as a membership function.

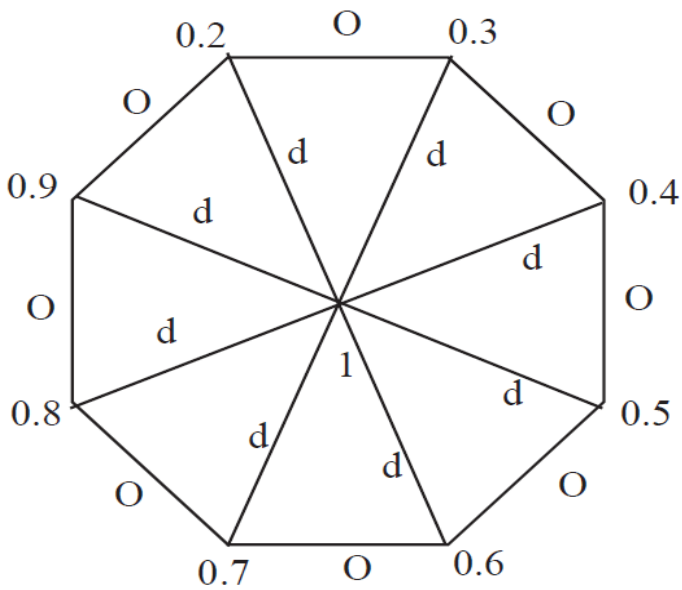
If 0 labels at the centre, then $1/10^i$ must be labeled at nowhere in the graph as a membership function.

Label the remaining vertex by any fuzzy membership function which can't be divided by one another except the centre vertex.

Therefore $|e_\mu(0)-e_\mu(d)| \leq 1$ holds.

**Illustration: 1.23**

Consider a wheel graph $W_9$.




Here $e_\mu(0) = 8$ , $e_\mu(d) = 8$

$|\ e_\mu(0) - e_\mu(d)\ | = |8 - 8| = |0| \leq 1$

Hence $|e_\mu(0) - e_\mu(d)| \leq 1$.

Therefore every wheel graph $W_n$ is a fuzzy divisor cordial graph.

## CONCLUSIONS

In this paper, we have discussed some mathematical inner beauty of fuzzy graphs. If we go in to the deep of cordial labeling of crisp graph, we can realize that for every crisp graph, we can't apply cordial labeling. However it is possible in fuzzy graphs. Especially in the view of fuzzy divisor cordial graph which playsa main role in this paper. Because |V| is finite also every vertex must be labeled in crisp graph. However in fuzzy graph, there are infinite number of chances to label the vertex from 0 to 1.

So clearly we can conclude that every fuzzy graphs and crisp graphs can be represented in the form of fuzzy divisor cordial graph. This paper is the strong gateway to prove the above statement.

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