EXPERIMENTATION AND THERMAL ANALYSIS OF CYLINDRICAL AND CONICAL SHAPED FINS

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ABSTRACT

The present project gives the thermal analysis of the cylindrical and conical profile of the fin by experimentally. The dimension of both fins is same. Length of the fin is 116 mm and diameter of the fin is 18 mm. In this project, we have chosen two profile of fin-like cylindrical, and conical. The material of fin is same for both profile which is alluminum alloy. Fins are prepared by CNC machine. We have conducted the experiment for free convection. We have found the temperature distribution for all the cases and after calculation of data, we have compared the heat transfer coefficient, heat transfer rate, Nusselt number (Nu), effectiveness and efficiency of both profile of the fin. We have found from an experiment, effectiveness is maximum for the cylindrical profile, but efficiency is lowest and in case conical efficiency is maximum but effectiveness is very less.

KEYWORDS: Cylindrical, Conical Profile, Experimentally Dimension

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INTRODUCTION

In many area of engineering application where energy transfers require the faster rate of heat transfer from a surface to ambient.

Due to this reasons for increasing the heat transfer rate very efficiently, heat exchanger part makes low weight, low volume, less cost and simpler shapes.

According to Newton’s law of cooling \( q = hA_s(T_s - T_a) \) suggest that convection heat transfer can be increased by three methods. The first method of increasing the heat transfer coefficient h. The second method of increasing the surface area \( A_s \). Last method by increasing the temperature difference \( (T_s - T_a) \). Film heat transfer coefficient h is a function of surface geometry, fluid flow rate, and its properties. And find out its optimum value is through control the all-governing parameters. Temperature differences between the surface and surrounding are limited by the surrounding temperature which depends on weather condition. A most attractive parameter which is the increase the convection heat transfer from the surface is surface area. It is exposed to surrounding by making protrusions on the solid structure. These protrusions are called as fins or extended surface or spines.

Various type fin profiles is shown in figure 1.
Fin or spines are generally found in the form of protrusions which is fitted to the surface of the base structure for the purpose of increase convection heat transfer from the base surface of the surrounding fluid.

Figure 1: Type of Fins: (a) Rectangular Spine (b) Triangular Spine (c) Concave Parabolic Spine (d) Convex Parabolic Spine (e) Annular Fin of Rectangular Profile (f) Cylindrical Tube with Fin of Annular Profile (g) Cylindrical Spine (h) Conical Spine (i) Spine with Convex Profile

In this paper two different profiles like cylindrical and conical taken and find the thermal performance of these fins.

Figure 2: Line Diagram of Cylindrical Fin (Spine)

Figure 3: Line Diagram of Conical Fin (Spine)
Material of Fin

A different types of material used in the making of the spine, like aluminum Al, copper Cu, mild steel Ms, brass, an alloy of aluminum etc. A Material of fin is required low cost, high thermal conductivity, low weight etc. Value of Thermal conductivity of Al is 205 w/MK, alloy of aluminium is 177 w/MK, Cu is 387 w/MK and Ms is 46 w/MK. We have selected alloy of aluminium as fin material because thermal conductivity is more, the cost is less and weight of fin are also low (less density).

Description of Setup

Figure 4 represents that the experimental apparatus used for experiments. It is consisting of temperature indicator, thermocouple, voltmeter, ammeter, dimmer stator, rectangular duct, heater and Base material (structure) in which fin attached are connected to the heater. The fins are attached across the rectangular duct. With the help of thermocouple and temperature indicator, temperature are measured at the different point. Length of fin is 116 mm and diameter of fin is 18 mm. one end of the fin is connected to the base material and the other end is exposed to ambient. Six thermocouples are used in this experiment, five are connected to fin and one is exposed to the atmosphere for measurement of the surrounding temperature. Dimmer stator is used to vary the power input to the heater.

Specification

Table 1

<table>
<thead>
<tr>
<th>Material of fin</th>
<th>Aluminium alloy</th>
</tr>
</thead>
<tbody>
<tr>
<td>Number of thermocouple</td>
<td>six thermocouple</td>
</tr>
<tr>
<td>Temperature indicator</td>
<td>digital, 0-300°C</td>
</tr>
<tr>
<td>Dimmer stator</td>
<td>2 Amp, 230 V</td>
</tr>
<tr>
<td>Heater</td>
<td>200 Watts Band type</td>
</tr>
<tr>
<td>Ammeter</td>
<td>digital, range 0 to 1.999 amp.</td>
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<tr>
<td>Voltmeter</td>
<td>digital, range 0 to 199.9 volts.</td>
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</table>

Figure 4: Experimentally Setup of Pin Fin
Equation of Fins

Assumption

Following Assumption are Used in Fin Equation

- Steady state heat transfer taking place only along the length of the fin (spine).
- Heat transfer coefficient on the faces of the spine is constant and its value is same on the whole surface of the spine.
- No heat generation within the fin.
- There is convective heat transfer at the tip of the fin.
- A material of spine is homogenous and isotropic.

Cylindrical Fin

Heat transfer rate through the cylindrical fin

\[ Q = \sqrt{h \pi k A \theta_0 \left[ \frac{\tanh \frac{mL}{h} + \frac{mL}{h}}{1 + \tanh \frac{mL}{h}} \right]} \]

Where fin parameter \( m = \left( \frac{\pi \beta h}{kA_\theta} \right) \)

Conical Fin

Heat transfer rate through the conical fin

\[ Q_{\text{conical}} = \frac{\pi kd^2 M \theta_0 J_2(2M\sqrt{d})}{4J_1(2M\sqrt{d})} \]

Where \( d \) is diameter of major end and \( M = \left( \sqrt{2m^2L} \right) \), \( m = \left( \frac{2A}{k \rho d} \right) \)

Experimental Data for Cylindrical Fin

<table>
<thead>
<tr>
<th>Time (Minute)</th>
<th>( T_1 ) (°C)</th>
<th>( T_2 ) (°C)</th>
<th>( T_3 ) (°C)</th>
<th>( T_4 ) (°C)</th>
<th>( T_5 ) (°C)</th>
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Table 2: Free Convection at I= 0.231 Amp, V=50 Volt

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CALCULATION

Cylindrical fin

Current = 0.231 amp, Voltage = 50 volt, Length of fin (L) = 0.116 m, Diameter of fin (d) = 0.018 m

Perimeter of the fin (p) = πd = 0.05654 m, Area of the fin (A) = πd²/4 = 2.54469 × 10⁻⁴ m²

When the steady condition achieved

Average temperature = \( T_{avg} = \frac{T_1 + T_2 + T_3 + T_4 + T_5}{5} = \frac{74.5 + 74.9 + 74.6 + 73.6 + 73.1}{5} = 74.3 \, ^\circ C \)

Ambient temperature = \( T_\infty = 31.6 \, ^\circ C \) (taking average)

Mean temperature of air = \( T_m = \frac{T_{avg} + T_c}{2} = \frac{74.3 + 31.6}{2} = 52.95 \approx 53 \, ^\circ C \)

The difference in temperature = \( \Delta T = T_{avg} - T_\infty = 74.3 - 31.6 = 42.7 \, ^\circ C \)
Properties of air at 53 °C (from HMT data book)

\[ \rho_a = 1.085 \text{ kg/m}^3, \mu = 19 \times 10^{-6} \text{ Ns/m}^2, Pr = 0.697, k_{air} = 0.02846 \text{ w/(mk)} \]

\[ \beta = 3.07 \times 10^{-3} (1/\text{k}) \] (HMT data book)

Grash of Number =

\[ \text{Gr} = \frac{\rho^2 g \beta \Delta T L^3}{\mu^2} = \frac{1.085^2 \times 9.81 \times 3.07 \times 10^{-3} \times 42.7 \times 1.16^3}{(19.75 \times 10^{-6})^2} = 6.058 \times 10^6 \]

GrPr = 4222426

Nusselt Number = Nu = 0.53(GrPr)\(^{(1/4)}\) = 24.025

Heat transfer coefficient =

\[ h = \frac{Nuk_{air}}{L} = \frac{24.025 \times 0.02846}{0.116} = 5.89 \text{ w/(m}^2\text{k)} \]

\[ Q = \sqrt{hpk_A \theta_0} \left[ \frac{\tanh ml + h}{1 + \tanh ml} \right] \text{ Fin parameter} = m = \sqrt{\frac{hp}{k_A}} = \sqrt{\frac{5.89 \times 0.05654}{177 \times 2.54469 \times 10^{-4}}} = 2.72 \]

\[ Q = 5.89 \times 0.05654 \times 177 \times 2.54469 \times 10^{-4} (75.5 - 31.6) = 1.762 \text{ w} \]

\[ Q_{\text{max}} = hA_f (T_b - T_\infty), \text{ Where } A_f \text{ surface area of fin including the tip} = \pi d L_c \]

\[ L_c = \text{ characteristic length of fin} = L + L/4 \]

\[ Q_{\text{max}} = 5.89 \times \pi \times 0.018 \times (0.116 + 0.018/4)(75.5 - 31.6) = 1.762 \text{ w} \]

\[ Q_{\text{without fin}} = hA_c (T_b - T_\infty) = 5.89 \times 2.54469 \times 10^{-4} (75.5 - 31.6) = 0.0658 \text{ w} \]

Efficiency = \[ \eta = \frac{Q}{Q_{\text{max}}} = \frac{1.762}{1.699} = 0.9642 = 96.42 \% \]

Effectiveness = \[ \epsilon = \frac{Q}{Q_{\text{without fin}}} = \frac{1.699}{0.0658} = 25.82 \]

**Conical Fin**

Current = 0.231 amp, Voltage = 50 volt, Length of fin (L) = 0.116, Diameter of fin (d) at major end 0.018 m

Area of fin (A_c) = \[ \pi d^2/4 = 2.54469 \times 10^{-4} \text{ m}^2 \]

When the steady condition achieved

Average temperature = \[ T_{avg} = \frac{T_1 + T_2 + T_3 + T_4 + T_5}{5} = \frac{85.3 + 85 + 84.6 + 83.6 + 82.9}{5} = 84.3 \text{ °C} \]

Ambient temperature = \[ T_\infty = 31.4 \text{ °C} \] (taking average)

Mean temperature of air = \[ T_m = \frac{T_{avg} + T_\infty}{2} = \frac{84.3 + 31.4}{2} = 57.85 \approx 57.8 \text{ °C} \]

The difference in temperature = \[ \Delta T = T_{avg} - T_\infty = 84.3 - 31.4 = 52.9 \text{ °C} \]

Properties of air at 57.8 °C (from HMT data book)

\[ \rho_a = 1.065 \text{ kg/m}^3, \mu = 20 \times 10^{-6} \text{ Ns/m}^2, Pr = 0.696, k_{air} = 0.02896 \text{ w/(mk)} \]
\[ \beta = 3.07 \times 10^{-3} \text{ (1/k)} \] (HMT data book)

Grashof Number \(= \text{Gr} = \frac{\rho g \beta \Delta T L^3}{\mu^2} = \frac{1.065^3 \times 9.81 \times 3.07 \times 10^{-3} \times 52.9 \times 1.16^3}{(20 \times 10^{-6})^2} = 6.96 \times 10^6 \]

GrPr = \(6.96 \times 10^6 \times 0.696 = 4844160\)

Nusselt Number = \(\text{Nu} = 0.53 \times \text{GrPr}^{\frac{1}{4}} = 24.865\)

Heat transfer coefficient = \(h = \frac{\text{Nu} \text{Re} \text{Pr}^2}{L} = \frac{24.865 \times 0.02096}{0.116} = 6.21 \text{ w/(m}^2\text{k)}\)

\[ Q = \frac{\pi d^2 \theta_0 L I_2(2M \sqrt{T})}{4 \sqrt{2 \pi}} \]

Where \(I_2 \) and \(I_1 \) first kind Bessel function second and first order respectively

Fin parameter = \(m = \frac{\sqrt{2h}}{kd} \)

Fin parameter = \(m = \frac{5.89 \times 2}{177 \times 0.018} = 1.974 \)

So \( M = (2 \times (1.974)^2 \times 0.116)^{1/2} = 0.95 \)

\[ Q = \frac{2 \times 177 \times 0.018 \times 0.95}{4 \sqrt{0.116}} \times (85.3 - 31.4) \times \frac{I_2(2 \times 0.95 \sqrt{0.116})}{I_1(2 \times 0.95 \sqrt{0.116})} = 6.772 \times \frac{I_2(0.65)}{I_1(0.65)} \]

From the Bessel table

\( I_1(0.65) = 0.34247 \)

\( I_0(0.65) = 1.10845 \)

\( I_2(x) = I_0(x) - \frac{2}{x} \times I_1(x) \)

\( I_2(0.65) = 0.0547 \)

\[ Q = 6.772 \times 0.0547 = 0.34247 = 1.081 \text{ w} \]

\[ Q_{\text{max}} = h \times \pi \times d/2 \times L \times (T_b - T_{\infty}) = 6.21 \times \pi \times 0.018/2 \times (0.116)^8(85.3 - 31.4) = 1.098 \text{ w} \]

\[ Q_{\text{without fin}} = h A_c (T_b - T_{\infty}) = 6.21 \times 2.54469 \times 10^{-4} \times (85.3 - 31.4) = 0.0852 \text{ w} \]

Efficiency = \(\eta = \frac{Q}{Q_{\text{max}}} = \frac{1.082}{1.098} = 0.9854 = 98.54 \% \)

Effectiveness = \(\varepsilon = \frac{Q}{Q_{\text{without fin}}} = \frac{1.082}{0.0852} = 12.92 \)

**RESULTS AND DISCUSSIONS**

<table>
<thead>
<tr>
<th>Shape</th>
<th>Nu</th>
<th>H</th>
<th>Q</th>
<th>(\eta)</th>
<th>(\varepsilon)</th>
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</table>
From the result the Nusselt number of a conical fin is more as the comparison of the Cylindrical then the heat transfer coefficient is also more in the conical than cylindrical because the Nusselt number more in conical fin case. The heat transfer is more in the case cylindrical fin as the comparison of the conical fin. An efficiency of fin maximum in case of conical but effectiveness is minimum as the comparison of a cylindrical fin.

**CONCLUSIONS**

The conclusion comes from the experimental investigation of different profile of the fin are bellow

- Heat transfer coefficient of fin: The value of heat transfer coefficient depends upon the cooling fluid and the geometry of the solid (fin). The maximum heat transfer coefficient is for conical fin and a minimum for the cylindrical fin.
- Fin effectiveness is maximum for cylindrical fin and the minimum for the conical fin. Effectiveness value of a cylindrical, and conical fin are 25.83, and 12.92 respectively in free convection.
- From the experimental result found that conical fin efficiency is maximum and lowest for the cylindrical fin. The efficiency of a conical and cylindrical fin is 98.54 and 96.42 respectively in free convection.

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