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SECONDARY FLOW OF TWO IMMISCIBLE LIQUIDS IN A ROTATING ANNULAR PIPE OF CIRCULAR CROSS SECTION

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ABSTRACT

Ramana Rao and Narayana (1981) studied the flow of two incompressible immiscible liquids occupying equal heights between two parallel plates in a rotating system under the action of constant pressure gradient. They also studied the associated thermal distribution, assuming equal and different plate temperatures. This branch of fluid mechanics has developed rapidly in recent times as an obvious consequence of interest in geophysical flow problems, earth's atmosphere, oceans and core and of stars and galaxies. We considered the flow of two incompressible immiscible fluids occupying equal heights in rotating circular pipe. Immiscible fluids we mean, superposed fluids of different densities and viscosities. The rotating pipe that we consider here has the following physical meaning. If we introduce a pipe in a rotating flow, for example, rotating flow due to earth's rotation, the pipe also rotates.

KEYWORDS: Pressure gradient, Flux

INTRODUCTION

This sets up the primary and secondary flows. Ramana Rao and Narayana (1981) suggested that olive oil and water can be taken as two immiscible liquids to test their theoretical conclusions for setting up an experiment.The uniqueness for two immiscible fluids in one-dimensional porous medium was studied by Baiocchi, Evans, Lawrence C, frank, Leonid, Friedman, Anver (1980). Patrudu (2001) studied the laminar flow of two incompressible immiscible liquids under a constant pressure gradient through a channel of circular cross section in a rotating straight pipe, rotating with a uniform angular velocity about an axis perpendicular to the channel. This problem was later extended by Sivarama Prasad (2006) for the hydro magnetic case.

The flow of two incompressible, immiscible liquids through a straight pipe in the annular region bounded by two concentric circles of radii εa and a, $\varepsilon < 1$, under two constant pressure gradients, occupying equal heights is considered.

BASIC EQUATIONS AND THEIR NON DIMENSIONAL FORM

We consider the steady laminar flow of two incompressible liquids under the action of constant pressure gradient through a channel of arbitrary cross section rotating with a uniform angular velocity about an axis perpendicular to the channel. The equations of motion in steady state flow, relative to a set of rectangular Cartesian coordinates $\bar{r}^1 = \bar{r}^1$ (x^1, y^1, z^1) rotating with a constant angular velocity Ω' with respect to an inertial system are

$$-2\rho_1\,\Omega' w_1 \sin\theta + \rho_1\left(u_1\frac{\partial u'_1}{\partial r'} + \frac{v'_1}{r'}\frac{\partial u'_1}{\partial \theta'} - \frac{{v'_1}^2}{r'}\right)$$

$$= -\frac{\partial \pi'}{\partial r'} + \rho_1 \nu_1\left(\nabla'^2 u'_1 - \frac{u'_1}{r'^2} - \frac{2}{r'^2}\frac{\partial v_1}{\partial \theta}\right) \quad (1.1)$$

$$-2\rho'_1\,\Omega' w_1 \cos\theta + \rho_1\left(u_1\frac{\partial u'_1}{\partial r'} + \frac{v'_1}{r'}\frac{\partial u'_1}{\partial \theta'} - \frac{u_1{v'_1}^2}{r'}\right)$$

$$= -\frac{1}{r'}\frac{\partial \pi'}{\partial r'} + \rho_1 \nu_1\left(\nabla'^2 u'_1 - \frac{v'_1}{r'^2} - \frac{2}{r'^2}\frac{\partial u_1}{\partial \theta}\right) \quad (1.2)$$

$$-2\rho'_1\,\Omega' u_1 \sin\theta + v'_1\cos\theta\left(u_1\frac{\partial w'_1}{\partial r'} + \frac{v'_1}{r'}\frac{\partial w'_1}{\partial \theta}\right)$$

$$= -\frac{\partial \pi'}{\partial z'} + \rho_1 \nu_1\,\nabla'^2\,w_1 \quad (1.3)$$

$$\frac{\partial u'}{\partial r'} + \frac{u'_1}{r'} + \frac{1}{r}\frac{\partial v'_1}{\partial \theta} = 0 \quad (1.4)$$

where $\pi' = P' - \frac{1}{2}\rho_1\,\Omega'^2\,(r'^2 \sin^2\theta + z'^2)$ (1.5)

$$\nabla'^2 = \frac{\partial^2}{\partial r^2} + \frac{1}{r'}\frac{\partial}{\partial r} + \frac{1}{r^2}\frac{\partial^2}{\partial \theta^2} \quad (1.6)$$

For fully developed laminar flow, the form of π' is restricted to

$\pi' = -cz' + F(r',\theta)$ (1.7)

where 'c' is a constant and may be termed as the gradient of π' along the axis of the pipe.

The above equations are the equations of motion of a viscous incompressible liquid characterized by viscosity ν_1 and density ρ_1 occupying the space between $r' = a \in$ and $r' = a$ in the circular pipe for the upper liquid. Here (u'_1, v'_1, w'_1) are the components of the velocity in the direction of (r', θ, z') where 'θ' is the angle between the radius and the axis of rotation and z^1 is measured from the axis of the pipe.

The corresponding equations for the lower liquid occupying the space between $r' = 0$ to $r' = a \in$ of the circular pipe, are

$$-2\rho_2\,\Omega' w_2 \sin\theta + \rho_2\left(u'_2\frac{\partial u'_2}{\partial r'} + \frac{v'_2}{r'}\frac{\partial u'_2}{\partial \theta'} - \frac{{v'_2}^2}{r'}\right)$$

$$= -\frac{\partial \pi'}{\partial r'} + \rho_2 \nu_2\left(\nabla'^2 u'_2 - \frac{u'_2}{r'^2} - \frac{2}{r'^2}\frac{\partial v_2}{\partial \theta}\right) \quad (1.8)$$

$$-2\rho'_2\,\Omega' w_2\cos\theta+\rho_2\left(u_2\frac{\partial v'_2}{\partial r'}+\frac{v'_2}{r'}\frac{\partial v'_2}{\partial\theta'}-\frac{u_2 v'_2}{r'}\right)$$

$$= -\frac{1}{r'}\frac{\partial\pi'}{\partial\theta}+\rho_2\nu_2\left(\nabla'^2 u'_2-\frac{v'_2}{r'^2}-\frac{2}{r'^2}\frac{\partial u_2}{\partial\theta}\right) \tag{1.9}$$

$$2\rho'_2\,\Omega'(u_2\sin\theta+v'_2\cos\theta+\rho_2\left(u_2\frac{\partial w'_2}{\partial r'}+\frac{v'_2}{r'}\frac{\partial w'_2}{\partial\theta}\right)$$

$$= -\frac{\partial\pi'}{\partial z'}+\rho_2\nu_2\,\nabla'^2 w_2 \tag{1.10}$$

$$\frac{\partial u'_2}{\partial r'}+\frac{u'_2}{r'}+\frac{1}{r}\frac{\partial v'_2}{\partial\theta}=0 \tag{1.11}$$

where $\pi' = P' - \frac{1}{2}\rho_2\,\Omega'^2\,(r'^2\sin^2\theta+z'^2)$ (1.12)

For the upper liquid, we introduce the stream function ϕ'_1 such that

$$r'u'_1 = \frac{\partial\phi'_1}{\partial\theta}$$

$$v'_1 = = \frac{\partial\phi'_1}{\partial r'} \tag{1.13}$$

where ϕ'_1 is a function of r' and θ only. Eliminating π' from (1.1)

and (1.2) we get

$$-2\Omega'(D*w_1)+\frac{1}{r}\partial\,(\phi'_1,\nabla'^2\phi'_1)=\nu_1\,\nabla'^4\,\phi_1 \tag{1.14}$$

Using eq. (1.13) in eq. (1.3), we get

$$-2\Omega'(D*\phi_1)+\frac{1}{r}\partial\,(\phi'_1,w'_1)=\nu_1\,\nabla'^2\,w_1 \tag{1.15}$$

where $D_* = \cos\theta\,\frac{\partial}{\partial r'}-\frac{\sin\theta}{r'}\frac{\partial}{\partial\theta}$ (1.16)

And ∂ (X, Y) stands for the Jacobian of X and Y with respect to r' and θ

respectively.

i.e. $J(X,Y)=\frac{\partial(X,\,Y)}{\partial(r',\theta)}$ (1.17)

For the lower liquid following similar analysis, we get

$$-2\Omega'(D*w_2)+\frac{1}{r}\partial\,(\phi'_2,\nabla'^2\phi'_2)=v_2\nabla'^4\phi_2 \quad (1.18)$$

$$2\Omega'(D*\phi_2)+\frac{1}{r}\partial\,(\phi'_2,w'_2)=\frac{c}{\rho_2}v_2\nabla'^2 w_2 \quad (1.19)$$

In terms of the non dimensional variables,

$$w'_1=\frac{c\,a^2 W_1}{4\rho_1 v_1},\qquad \phi_1=\frac{c\,a^3\phi_1}{4\rho_1 v_1},$$

$$R'_1=\frac{c\,a^3}{4\rho_1 v^2{}_1},\qquad T_1=\frac{2\Omega' a^2}{v_1}$$

(1.20)

$$w'_2=\frac{c\,a^2 W_2}{4\rho_2 v_2},\qquad \phi_2=\frac{c\,a^3\phi_2}{4\rho_2 v_2},$$

$$R'_2=\frac{c\,a^3}{4\rho_1 v^2{}_2},\qquad T_2=\frac{2\Omega' a^2}{v_2}$$

where ‘c’ is the pressure gradient, R_1, R_2 stand for the Reynolds numbers of the upper and lower liquids respectively, T_1 and T_2 are the Taylor numbers for these liquids respectively, we get for

Upper Liquid

$$T_1\;(D*\phi_1)+\frac{R}{r}\partial\,(\phi'_1,w'_1)=4+\nabla^2 w_1 \quad (1.21)$$

$$-T_1\;(D*W_1)+\frac{R}{r}\partial\,(\phi'_1,\nabla^2\phi_1)=\nabla^4\phi_1 \quad (1.22)$$

Lower Liquid

$$T_2\;(D*\phi_2)+\frac{R_2}{r}\partial\,(\phi'_2,w'_2)=4+\nabla^2 w_2 \quad (1.23)$$

$$-T_2\;(D*W_2)+\frac{R_2}{r}\partial\,(\phi'_2,\nabla^2\phi_2)=\nabla^4\phi_2 \quad (1.24)$$

where $D_* = \cos\theta\,\frac{\partial}{\partial r} - \frac{\sin\theta}{r}\frac{\partial}{\partial\theta}$ (1.25)

and when k = 1 the pressure gradient is same for both the liquids. For different values of k, the pressure gradient is varied.

Boundary Conditions

To find the solutions for W_1, ϕ_1, W_2, ϕ_2 we use the following boundary conditions given in non dimensional form, at r = 1, $W_1 = 0$ and $\frac{\partial \phi_1}{\partial r} = \phi_1 = 0$ (1.26)

at r = $\in$, $W_2 = 0$ and $\frac{\partial \phi_2}{\partial r} = \phi_2 = 0$ (1.27)

at r = b, (line between $\in$ and 1 i.e. $\in \leq b \leq 1$)

$$-\frac{1}{r}\frac{\partial \phi_1}{\partial r} = -\frac{1}{r}\frac{\partial \phi_2}{\partial r} \Rightarrow \frac{\partial \phi_1}{\partial r} = \frac{\partial \phi_2}{\partial r},\ W_1 = W_2$$

$$\frac{\partial W_1}{\partial r} = L\frac{\partial W_2}{\partial r}\ where\ L = \frac{\mu_2}{\mu_1} \qquad (1.28)$$

Method of Solution

For the upper liquid, we assume

$$\phi_1 = T_1\phi_{11} + T_1^2\phi_{12} + \ldots\ldots \qquad (1.29)$$

$$W_1 = W_{10} + T_1W_{11} + T_1^2 W_{12} + \ldots.. \qquad (1.30)$$

For the lower liquid, we assume

$$\phi_2 = T_2\phi_{21} + T_2^2\phi_{22} + \ldots\ldots \qquad (1.31)$$

$$W_2 = W_{20} + T_2W_{21} + T_2^2 W_{22} + \ldots.. \qquad (1.32)$$

Solutions

Substituting equations (1.29) and (1.30) in equation (1.21) & equations (1.31) and (1.32) in equation (1.23) and to the zeroth power in both T_1 and T_2, we get

$$\nabla^2 W_{10} = -4 \qquad (1.33)$$

$$\nabla^2 W_{20} = -4K \qquad (1.34)$$

which are to be solved with the appropriate boundary conditions to this approximation, namely

$W_{10} = 0$ at r = 1

$W_{10} = W_{20}$ at r = ε

$$\frac{\partial W_{10}}{\partial r} = L\frac{\partial W_{20}}{\partial r}\ at\ r = b$$

and $W_{20} = 0$ at r = $\in$ (1.35)

On solving eqs. (1.33) and (1.34), we get

$$W_{10} = C_1 + C_2 \log r - r^2 \qquad (1.36)$$

$$W_{20} = C_3 + C_4 \log r - kr^2 \qquad (1.37)$$

where

$$C_4 = \frac{1+b^2\,(2\log b-1)-kb^2\,(2L\log b-1)-k\in^2}{(1-L)\log b-\log\in}$$

$C_3 = k\in^2 - C_4 \log \in$

$C_2 = LC_4 - 2kLb^2 + 2b^2$

$C_1 = 1$

To the first order approximation in T_1 and T_2 , using the expressions (1.22),

(1.24), (1.36) and (1.37) for w_{10} and w_{20} we get

for the upper liquid

$$\nabla^4 \phi_{11} = -D_* W_{10} \qquad (1.38)$$

and for the lower liquid

$$\nabla^4 \phi_{21} = -D_* W_{20} \qquad (1.39)$$

where $D_* = \cos\theta \dfrac{\partial}{\partial r} - \dfrac{\sin\theta}{r}\dfrac{\partial}{\partial\theta}$

$$D_* W_{10} = C_2 \frac{Cos\theta}{r} - 2r\cos\theta \qquad (1.40)$$

$$D_* W_{20} = C_4 \frac{Cos\theta}{r} - 2kr\cos\theta \qquad (1.41)$$

These are to be solved subject to the conditions

at $r = 1$, $W_{11} = 0$, $\varphi_{11} = 0$ and $\dfrac{\partial\phi_{11}}{\partial r} = 0$

at $r = \in$, $W_{21} = 0$, $\varphi_{21} = 0$ and $\dfrac{\partial\phi_{21}}{\partial r} = 0$

at $r = b$, $-\dfrac{1}{r}\dfrac{\partial\phi_{11}}{\partial r} = -\dfrac{1}{r}\dfrac{\partial\phi_{21}}{\partial r}$

$$\Rightarrow \frac{\partial\phi_{11}}{\partial r} = \frac{\partial\phi_{21}}{\partial r} \quad W_{11} = W_{21}, \quad \frac{\partial W_{11}}{\partial r} = L\frac{\partial W_{21}}{\partial r} \qquad (1.42)$$

and writing

$\varphi_{11} = g_{11}(r)\cos\theta$

$$\varphi_{21} = g_{21}(r)\cos\theta \qquad (1.43)$$

in eqs. (1.38), (1.39) we get

$$g_{11} = \frac{1}{192}\left[A_1 r + A_2 r\log r + \frac{A_3}{r} + A_4 r^3 + 2r^5 - 12c_2 r^3 \log r\right] \quad (1.44)$$

$$g_{21} = \frac{1}{192}\left[A_5 r + A_6 r\log r + \frac{A_7}{r} + A_8 r^3 + 2kr^5 - 12c_4 r^3 \log r\right] \quad (1.45)$$

$$A_4 = \frac{P_3 P_4 - P_1 P_6}{P_2 P_4 - P_1 P_5}$$

$$A_3 = \frac{P_3 P_5 - P_2 P_6}{P_2 P_5 - P_2 P_5}$$

$A_2 = 12C_2 - 8 + 2A_3 - 2A_4$

$A_1 = -2 - A_3 - A_4$

$P_1 = 2b \log b + \frac{1}{b} - b$

$P_2 = b^3 - b - 2b \log b$

$P_3 = 12C_2 b^3 \log b - 2b^5 + 2b - 12 C_2 b \log b + 8b \log b$

$P_4 = 1 + 2 \text{ lob } b - \frac{1}{b^2}$

$P_5 = -3 - 2 \log b + 3b^2$

$P_6 = 36C_2 b^2 \log b + 12C_2 b_2 - 10b^4 + 10 - 12C_2 - 12C_2 \log b + 8 \log b$

$$A_5 = \frac{p_9 p_{11} - p_8 p_{12}}{p_7 p_{11} - p_8 p_{10}}$$

$$A_6 = \frac{p_9 p_{10} - p_7 p_{12}}{p_8 p_{10} - p_7 p_{11}}$$

$$A_7 = b[12C_4 b^3 \log b - 2kb^5 - A_5 b - A_6 b \log b - A_8 b^3]$$

$$A_8 = \frac{\{12C_4 \in^4 \log \in + 36C_4 b^4 \log b + 12C_4 b^4 - 10kb^4 - 2k \in^6 - A_5(\in^2 + b^2) - A_6(b^2 + b^2 \log b + \in^2 \log b)\}}{\in^4 + 3b^4}$$

$$p_7 = 2\in^2 b^4 - 2\in^4 b^2$$

$$p_8 = \in^2 b^4 - \in^4 b^2 + 2\in^2 b^4 \log \in - 2\in^4 b^2 \log b$$

$$p_9 = 48C_4 \in^4 b^4 \log \in - 48C_4 \in^4 b^4 \log b + 12k \in^6 b^4 + 12k \in^4 b^6$$

$$p_{10} = -2\in^6 +6\in^2 b^4 - 4\in^4 b^2$$

$$p_{11} = \in^6 -2\in^6 \log\in +3\in^2 b^4 + 6\in^2 b^4 \log\in -4\in^4 b^2 - 4\in^4 b^2 \log b$$

$$p_{12} = 144C_4 \in^4 b^4 \log(b\in) - 12C_4 \in^4 b^4 - 36k\in^6 b^4 + 40k\in^4 b^4 - 4C_4 \in^{10} +12C_4 \in^8$$

To the first order of approximation in T_1 and T_2 from equations (1.21) and (1.23), we get

$$\nabla^2 w_{11} = \frac{R_1}{r}\partial(\phi_{11}, w_{10}) \tag{1.46}$$

$$\nabla^2 w_{21} = \frac{R_2}{r}\partial(\phi_{21}, w_{20}). \tag{1.47}$$

Using the expressions for $\phi_{11}, \phi_{21}, w_{10}, w_{20}$ from equations (1.43), (1.40) and (1.41) in eqs. (1.46), (1.47) we get

$$\nabla^2 w_{11} = \frac{R_1}{r^2}\left\{\left(C_2 - 2r^2\right)g_{11}(r)\sin\theta\right\} \tag{1.48}$$

$$\nabla^2 w_{21} = \frac{R_2}{r^2}\left\{\left(C_4 - 2kr^2\right)g_{21}(r)\sin\theta\right\} \tag{1.49}$$

The boundary conditions on w_{11} and w_{21} are

at $r = 1$, $w_{11} = 0$

at $r = b$, $w_{11} = \lambda w_{21}$, $\dfrac{\partial w_{11}}{\partial r} = \lambda L \dfrac{\partial w_{21}}{\partial r}$

at $r = \in$, $w_{21} = 0$, (1.50)

and $\lambda = \dfrac{T_2}{T_1}$.

Solving equations (1.48) and (1.49), we get

$w_{11} = h_{11}(r)\sin\theta$

$w_{21} = h_{21}(r)\sin\theta$

where (1.51)

$$h_{11}(r) = A_9 r + \frac{A_{10}}{r} + \frac{R_1}{96}\{A_{11} r\log r + A_{12} r^3 \log r + A_{13} r^5 \log r + A_{14} r(\log r)^2 + A_{15}\frac{\log r}{r} + A_{16} r^3 + A_{17} r^5 + A_{18} r^7\} \tag{1.52}$$

$$h_{21}(r) = A_{19}\, r + \frac{A_{20}}{r} + \frac{R_2}{96}\{A_{21} r\log r + A_{22} r^3 \log r + A_{23} r^5 \log r +$$

$$A_{24} r\,(\log r)^2 + A_{25}\frac{\log r}{r} + A_{26} r^3 + A_{27} r^5 + A_{28} r^7\} \tag{1.53}$$

and

$$A_9 = -p_{13} - A_{10}$$

$$A_{10} = \frac{p_{14} \in - p_{13} \in^2 + A_{20}}{\in^2}$$

$$A_{19} = \frac{p_{21} p_{23} - p_{20} p_{24}}{p_{19} p_{23} - p_{20} p_{22}}$$

$$A_{20} = \frac{p_{21} p_{22} - p_{19} p_{24}}{p_{20} p_{22} - p_{19} p_{23}}$$

$$p_{13} = \frac{R_1}{96}\{A_{16} + A_{17} + A_{18}\}$$

$$p_{14} = \frac{R_2}{96}\{A_{21} \in \log \in + A_{22} \in^3 \log \in + A_{23} \in^5 \log \in + A_{24} \in (\log \in)^2 + A_{25}\frac{\log \in}{\in} + A_{26} \in^3 + A_{27} \in^5 + A_{28} \in^7\}$$

$$p_{15} = \frac{R_1}{96}\{A_{11} b\log b + A_{12} b^3 \log b + A_{13} b^5 \log b + A_{14} b(\log b)^2 + A_{15}\frac{\log b}{b} + A_{16} b^3 + A_{17} b^5 + A_{18} b^7\}$$

$$p_{16} = \lambda\frac{R_2}{96}\{A_{21} b\log b + A_{22} b^3 \log b + A_{23} b^5 \log b + A_{24} b(\log b)^2 + A_{25}\frac{\log b}{b} + A_{26} b^3 + A_{27} b^5 + A_{28} b^7\}$$

$$p_{17} = \frac{R_1}{96}\{A_{11} + A_{11}\log b + A_{12} b^2 + 3A_{12} b^2 \log b + A_{13}(b^4 + 5b^4 \log b) + A_{14}((\log b)^2 + 2\log b) + A_{15}\left(\frac{1}{b^2} - \frac{\log b}{b^2}\right)$$
$$+3A_{16} b^2 + 5A_{17} b^4 + 7A_{18} b^6\}$$

$$p_{18} = \lambda L\frac{R_2}{96}\{A_{21} + A_{21}\log b + A_{22} b^2 + 3A_{22} b^2 \log b + A_{23}(b^4 + 5b^4 \log b) + A_{24}((\log b)^2 + 2\log b) + A_{25}\left(\frac{1}{b^2} -\right.$$
$$\left.\frac{\log b}{b^2}\right) + 3A_{26} b^2 + 5A_{27} b^4 + 7A_{28} b^6\}$$

$$p_{19} = -\lambda b^2 \in^2$$

$$p_{20} = 1 - b^2 - \lambda \in^2$$

$$p_{21} = (p_{16} - p_{15}) b \in^2 - p_{14} \in + p_{14} b^2 \in + p_{13} \in^2$$

$p_{22} = -\lambda\, Lb^2 \in^2$

$p_{23} = -1 - b^2 - \lambda\, L \in^2$

$p_{24} = (p_{18} - p_{17})b^2 \in^2 + p_{14}b^2 \in + p_{14} \in - p_{13} \in^2$

$A_{11} = \frac{A_1C_2}{4} - \frac{A_3}{2} - \frac{A_2C_2}{8}$, $A_{12} = \frac{-A_2}{8} - \frac{3C_2^2}{4}$, $A_{13} = \frac{C_2}{2}$,

$A_{14} = \frac{A_2C_2}{8}$, $A_{15} = \frac{-A_3C_2}{4}$, $A_{16} = \frac{2A_4C_2 - 4A_1 + 3A_2 + 18C_2^2}{32}$, $A_{17} = \frac{-C_2}{6} - \frac{A_4}{24}$

$A_{18} = \frac{-1}{24}$, $A_{21} = \frac{2C_4A_5 - 4kA_7 - C_4A_6}{8}$, $A_{22} = \frac{-6C_4^2 - kA_6}{8}$, $A_{23} = \frac{kC_4}{2}$, $A_{24} = \frac{C_4A_6}{8}$

$A_{25} = \frac{-C_4A_7}{4}$, $A_{26} = \frac{2C_4A_8 - 4kA_5 + 18C_4^2 + 3kA_6}{32}$, $A_{27} = \frac{-kA_8 - 4kC_4}{24}$, $A_{28} = \frac{-k^2}{24}$

DISCUSSIONS OF THE STREAMLINES IN THE CENTRAL PLANE

In the central plane perpendicular to the axis of rotation $\theta = \frac{\pi}{2}$ or $\frac{3\pi}{2}$, it can be seen that from the equations (1.43), v' = 0 in either case. So a particle of liquid once in this plane does not leave it in the subsequent motion. The motion in the two halves of the pipe is therefore quite distinct from each other.

The differential equation of the streamlines in the central plane of the pipe is

$$\frac{dy^1}{du^1} = \frac{dz^1}{dw^1} \quad \left(\theta = \frac{\pi}{2} \text{ or } 3\frac{\pi}{2}\right) \tag{1.54}$$

$$dz^1 = \frac{w^1 dr^1}{u^1} \quad where\ u^1 = -\frac{1}{r^1}\frac{\partial\phi^1}{\partial\theta} \tag{1.55}$$

$$\therefore dz^1 = \frac{w_2}{-\frac{1}{r^1}\frac{\partial\phi^1}{\partial\theta}} \quad where\ \phi_{21} = g_{21}(r)\cos\theta \tag{1.56}$$

$$T_2 dz = \frac{rw_2 dr}{-g_{21}(-\sin\theta)} = \frac{rw_2 dr}{g_{21}\sin\theta}$$

$$= \frac{r(w_{20} + T_2 w_{21})dr}{g_{21}\sin\theta} \tag{1.57}$$

$$= \frac{r(w_{20} + T_2 w_{21})dr}{\frac{1}{192r}(A_5 r^2 + A_6 r^2 \log r + A_8 r^4 + 2kr^6 - 12c_4 r^4 \cos\theta)}$$

$$= \frac{192r^2 (w_{20} + T_2 w_{21})dr}{(A_5 r^2 + A_6 r^2 \log r + A_8 r^4 + 2kr^6 - 12c_4 r^4 \cos\theta)}$$

To a sufficient approximation these streamlines for the lower liquid are given by

$$T_2 (z^1 - z_0^1) = \frac{192}{A_7}[l_1 r^3 + l_2 r^5 + l_3 - l_4]$$

where

$$l_1 = \frac{3k\in^2 - 3c_4\in^2 \log\in - c_4}{9} \tag{1.58}$$

$$l_2 = \frac{-5kA_7 - 5kA_5\in^2 + 5kA_5 c_4 \log\in + A_5 c_4 + kA_6\in^2}{25A_7}$$

$$l_3 = \frac{c_4 r^3 \log r}{3}$$

$$l_4 = \frac{A_5 c_4 r^3 \log r}{5A_7}$$

and $z_o^{'}$ is a constant of integration which is different for different stream lines and equal to the distance of the point r=0 on the curve from the axis of rotation.

Values of $T_1(z^1 - z_0^1)$ for $\in$ = 0.5 b = 0.75

r	$T_1(z^1 - z_0^1)$
0.75	0.19313
0.8	1.11275
0.85	2.64434
0.90	5.00022
0.95	8.43429
1.0	0

Values of $T_2(z^1 - z_0^1)$ for $\in$ = 0.5 b = 0.75

r	$T_2(z^1 - z_0^1)$
0.1	0.01633
0.2	0.10848
0.3	0.26647
0.4	0.52794
0.5	0.87765

To a sufficient approximation these streamlines for the upper liquid are given by

$$T_1(z^1 - z_0^1) = \frac{192}{A_3}\left[\frac{r^3}{3} + l_5 r^5 + \frac{c_2}{3} r^3 \log r + l_6 r^3 \log r\right] \quad (1.59)$$

where

$$l_5 = \left(\frac{-A_1 - A_3}{5A_3}\right)$$

$$l_6 = \left(\frac{-A_1 c_2 - A_2}{5A_2}\right)$$

Table shows the same stream line in the plane of symmetry for ε =0 and ε =1. We note that no stream line in the central plane ever reach the edge of the pipe. As the angular velocity Ω' is increased, the distance which must be covered by the central stream line to be within a given distance from the edge gets smaller., this results holds good for all values of ε between 0.5 and 1.

For a fixed value of T_2, the effect of decreasing ε from unity to 0.5, is to increase the distance that the liquid particle in the central plane travel in going from points near edge of the pipe i.e., r=0 to points r= ε. The similar conclusions can be drawn for the upper liquid also.

STREAM LINES OF THE SECONDARY FLOW

To the first order of approximation in T_1 and T_2, the equation for the projection of the stream lines on the cross section of the channel for the upper liquid is given by 192000 $g_{11}\cos\theta = k$.

Figure gives the projection of the stream lines on the cross section of the channel when $\in = 0.5$ for some values of k.

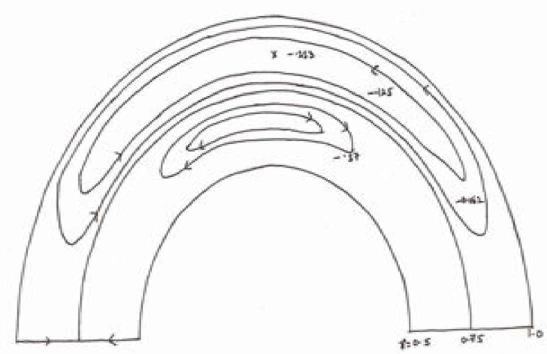

Fig 2.1 PROJECTION OF THE STREAM LINES ON THE UPPER CROSS SECTION OF THE CHANNEL

CONCLUSIONS

The stream line pattern in the central plane of the pipe has been discussed and the pattern of general stream lines for both upper and lower liquids have been obtained and shown graphically also. For different values of k, we get a set of closed curves which are symmetrical about the axis of rotation θ= 0. For an equal and opposite value of k, the curves in the lower half are obtained by the reflection of the curves on the diameter of the cross section perpendicular to the axis of rotation. The upper half of the cross section is divided into two regions by a circle of radius $r = r^*$, which corresponds to a stream line when k= 0. The other branches of the curve with k= 0 are the peripheries of the channel r=ε, r= 1 and the parts of the axis $\theta = \pm\frac{\pi}{2}$ between two peripheries. In the region I ($\varepsilon \le r \le r^*$), the stream lines are obtained as k assumes negative values between 0 and minimum value k^1(corresponding to the degenerate stream line for $r = r_{min}$). In the region II ($r^* \le r \le 1$), the stream lines are obtained as k assumes positive values between 0 and maximum value k^{11}(corresponding to the degenerate stream lines for $r = r_{max}$). At these degenerate points, both the radial and transverse components of velocity in the cross section vanish. Thus the stream lines of motion through the points (r_{min}, 0), (r_{max}, 0), (r_{min}, π), (r_{max}, π) are straight lines and the motion in opposite directions about two pairs of straight lines. It is found for ε= 0.5, r_{min} = 0.79, k^1= -0.02404, r^* = 0.87, r_{max} = 0.96, k^{11} = 0.00003. The figure gives the projection of the stream lines on the cross section of the channel when ε= 0.5 for some values of k. Similarly for the lower liquid it is found that for ε= 0.5, r_{min} = 0.0.64, k^1= -0.02404, r^* = 0.523, r_{max} = 0.74, k^{11} = 0.00003.

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