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KINEMATICS OF TWO BODIES IN TERMS OF GEOMETRIC SERIES

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ABSTRACT

In this paper the motion of two bodies moving along straight lines with uniform velocities have been considered and studied with the help of Geometric series. When two bodies move in same straight line with different uniform velocities then the distance between them varies continuously and follows a geometric progression. Three different cases have been discussed in this paper. In first case both the bodies move along a straight line in same direction and in the second case bodies move along parallel lines and in third case they move along non-parallel lines.

KEYWORDS: Two Bodies**,** Straight, Geometric Series

INTRODUCTION

Geometric progression is the progression in which every term is a multiple of its previous term. The ratio of two consecutive terms in a constant and is known as common ratio. Following is an example of Geometric progression:

$$a, a\,r, ar^2, ar^3 \_\_\_\_\_\_\_\_\_\_ ar^{n-1}$$

where n^{th} term is ar^{n-1}. **'r'** is the common ratio of that geometric progression. A Geometric series is given by

$$a + a\,r + ar^2 \_\_\_\_\_\_\_\_\_\_ ar^{n-1}$$

The sum of 1^{st} n geometric series is

$$S_n = \frac{a(1 - r^n)}{1 - r}$$

If **r >1** then it will be an increasing geometric series and if $0 < r < 1$ then it will be a decreasing geometric series the sum of infinite decreasing geometric series is

$$S = \frac{a}{1 - r}$$

RESULTS AND DISCUSSIONS

The Kinematics of Two Bodies in Terms of Geometric Series

CASE 1 (When the Bodies are Moving in a Same Straight Line and Same Direction)

Let body 'A' and 'B' are at rest and the distance between them is 'd_1'. Both bodies are placed in a straight line. Body 'A' is at the front and body 'B' is at the back. In the same time both the bodies start moving with uniform velocities. Body 'A' start moving with velocity 'U' and body 'B' start moving with velocity 'V'. Body 'B' takes time 't_1' to travel distance 'd_1' and reaches to the 1^{st} position of body 'A' and in time 't_1' body 'A' travels distance 'd_2'. Then body 'B' takes time 't_2'to travel distance 'd_2' and reaches the 2^{nd} position of body 'A'. In the same time body 'A' travels distance 'd_3' then body 'B' takes time 't_3'to travel distance 'd_3' and reaches the 3^{rd} position of body 'A'. In the same time body 'A' travels distance 'd_4' and so on. Follow figure 1 and figure 2.

$\mathbf{t_1 = \frac{d_1}{v}}$

$\mathbf{d_2 = u\, t_1, \text{ where } t_1 = \frac{d_1}{v}}$

$\mathbf{d_2 = \frac{u}{v} d_1}$

$\mathbf{t_2 = \frac{d_2}{v}, \text{ where } d_2 = \frac{u}{v} d_1}$

$\mathbf{t_2 = \frac{d_1}{v} . \frac{u}{v}}$

$\mathbf{d_3 = u\, t_2, \text{ where } t_2 = \frac{d_1}{v} . \frac{u}{v}}$

$\mathbf{d_3 = \left(\frac{u}{v}\right)^2 . d_1}$

$\mathbf{t_3 = \frac{d_3}{v}, \text{ where } d_3 = \left(\frac{u}{v}\right)^2 . d_1}$

A_2 A_3

$\mathbf{t_3 = \frac{d_1}{v} . \left(\frac{u}{v}\right)^2}$

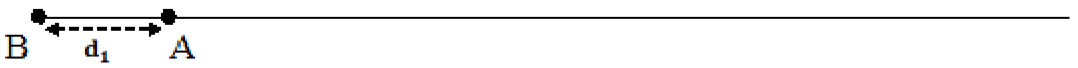

Figure 1: Initially the Bodies are at Rest and at their 1st Position

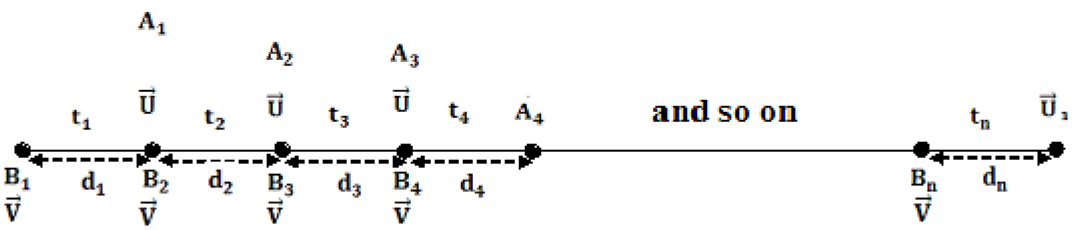

Figure 2: In Same Time Both the Bodies Start Moving with Uniform Velocities

$\mathbf{d_4 = u\, t_3, \text{ where } t_3 = \frac{d_1}{v} . \left(\frac{u}{v}\right)^2}$

$\mathbf{d_4 = \left(\frac{u}{v}\right)^3 . d_1}$

$\mathbf{t_1 = \frac{d_1}{v}, t_2 = \frac{d_1}{v} \frac{u}{v}, t_3 = \frac{d_1}{v} . \left(\frac{u}{v}\right)^2 \text{ \_\_\_\_\_\_\_\_\_\_\_\_ } t_n = \frac{d_1}{v} . \left(\frac{u}{v}\right)^{n-1}}$

It means that all the times from '$\mathbf{t_1}$'to '$\mathbf{t_n}$'are in geometric progression so we can apply all the formulas of geometric series in it.

Also,

$\mathbf{d_1, d_2, d_3, \text{\_\_\_\_\_\_\_\_} d_n}$ are in geometric progression so we can apply all the formulas of geometric series in it.

If velocity of body 'B' is greater than the velocity of body 'A'**(V > U)** then in time **'T'** body 'B' will collide with body 'A'. Then we know that $\mathbf{T = \frac{d_1}{V-U}}$ where **(V- U)** is the relative velocity of body 'B' with respect to body 'A'. Kinematics of two bodies in terms of geometric series also explain it

We know that if **V > U** then

$\mathbf{d_1, d_2, \_\_\_\_\_\_ d_n}$ are in decreasing geometric progression. Also $\mathbf{t_1, t_2, t_3 \_\_\_\_\_\_\_\_ t_n}$ are in decreasing geometric progression.

Also the time in which 'B' will collide with 'A' is the sum of all the times from t_1 to infinity

$$\mathbf{T = t_1+t_2+t_3 \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_}$$

Also we know that the sum of geometric series $\mathbf{a + ar + ar^2 \_\_\_\_\_\_\_\_\_\_\_}$ to infinity is

$$\mathbf{S = \frac{a}{1-r}}$$

Where $\mathbf{S = T,\ a = t_1 = \frac{d_1}{V},\ r = \frac{t_2}{t_1} = \frac{u}{V}}$

Substitute these values in above equation

$$\mathbf{T = \frac{\frac{d_1}{V}}{1-\left(\frac{U}{V}\right)}}$$

$\mathbf{T = \frac{d_1}{V-U}}$ **<u>Proved.</u>**

(CASE 2) When the Bodies are Moving in Parallel Lines

Supposed body 'A' is placed in line $\mathbf{'L_1'}$ and Body 'B' is placed in line $\mathbf{'L_2'}$ as shown. $\mathbf{'L_1'}$ and $\mathbf{'L_2'}$ are parallel to each other and the smallest and perpendicular distance between $\mathbf{'L_1'}$ and $\mathbf{'L_2'}$ is d_y. Distance between 'A' and 'B' is $\mathbf{d_1 = \sqrt{{d_{1x}}^2 + {d_y}^2}}$ as shown in figure 3. In the same time both the bodies start moving with uniform velocities. Body 'A' start moving with velocity 'U' and 'B' start moving with velocity 'V'. In time $\mathbf{'t_1'}$ body 'B' travels distance 'd_{1x}' and body 'A' travels distance 'd_{2x}'. Now 'A' and 'B' are in their 2^{nd} position and distance between them is 'd_2'. Then in time $\mathbf{'t_2'}$ body 'B' travels distance 'd_{2x}' and 'A' travels distance 'd_{3x}'. Now 'A' and 'B' are in their 3^{rd} position and the distance between them is 'd_3' and so on follow figure '3' and figure '4'.

$$\mathbf{d_1 = \sqrt{{d_{1x}}^2 + {d_y}^2}}$$

$$\mathbf{t_1 = \frac{d_{1x}}{V}}$$

$$\mathbf{d_{2x} = u\ t_1}$$

Where $\mathbf{t_1 = \frac{d_{1x}}{V}}$

$$\mathbf{d_{2x} = d_{1x} . \left(\frac{U}{V}\right)}$$

$$\mathbf{d_2 = \sqrt{{d_{2x}}^2 + {d_y}^2}}$$

$$\mathbf{d_2 = \sqrt{\left(d_{1x} . \frac{U}{V}\right)^2 + {d_y}^2}}$$

$$\mathbf{t_2 = \frac{d_{2x}}{V}}$$

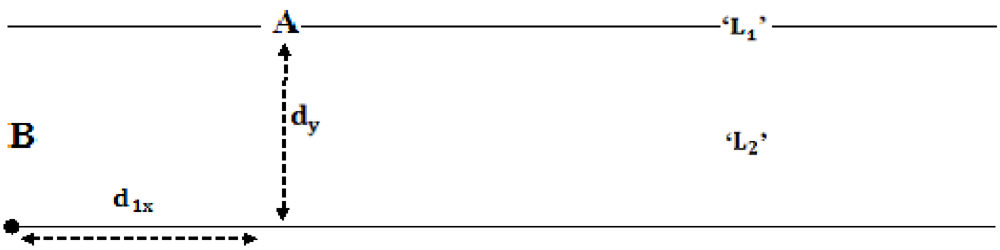

Figure 3: Initially the Bodies are at Rest and at their 1st Position

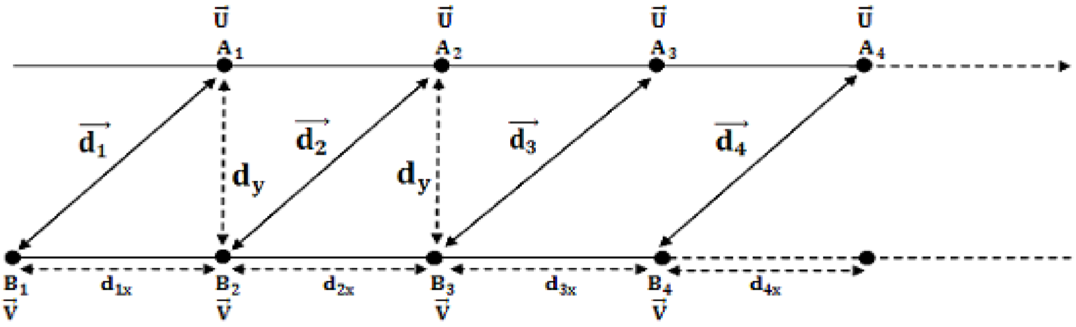

Figure 4: In Same Time Both the Bodies Start Moving with Uniform Velocities

Where $d_{2x}=d_{1x}\cdot\left(\frac{U}{V}\right)$

$t_2 = \frac{d_{1x}}{V}\cdot\left(\frac{U}{V}\right)$

$d_{3x} = U\, t_2$

$d_{3x} = U\,\frac{d_{1x}}{V}\cdot\left(\frac{U}{V}\right)$

$d_{3x}= d_{1x}\cdot\left(\frac{U^2}{V^2}\right)$

$d_3 = \sqrt{{d_{3x}}^2 + {d_y}^2}$

$d_3 = \sqrt{\left(d_{1x}\cdot\frac{U^2}{V^2}\right)^2 + {d_y}^2}$

$t_3 = \frac{d_{3x}}{V}$ where $d_{3x}= d_{1x}\cdot\left(\frac{U^2}{V^2}\right)$

$t_3 =\frac{d_{1x}}{V}\cdot\left(\frac{U^2}{V^2}\right)$

$d_{4x} = U\, t_3$

$d_{4x} = U.\,\frac{d_{1x}}{V}\cdot\left(\frac{U^2}{V^2}\right)$

$d_{4x}= d_{1x}\cdot\left(\frac{U^3}{V^3}\right)$

$d_4 = \sqrt{{d_{4x}}^2 + {d_y}^2}$

$d_4 = \sqrt{\left(d_{1x}\cdot\frac{U^3}{V^3}\right)^2 + {d_y}^2}$

t_1, t_2, t_3________________, t_n

$\frac{d_{1x}}{V}, \frac{d_{1x}}{V}.\frac{U}{V}, \frac{d_{1x}}{V}.\frac{U^2}{V^2}, \_\_\_\_\_\_\_\_\_\_\_\_ \frac{d_{1x}}{V}\left(\frac{U}{V}\right)^{n-1}$

$\frac{t_2}{t_1} = \frac{t_3}{t_2} = \frac{t_{n+1}}{t_n} = \frac{u}{V}$

It means that $t_1, t_2, \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_ t_n$ are in geometric progression.

Therefore, $\mathbf{t_n} = \frac{d_{1x}}{V} . \left(\frac{U}{V}\right)^{n-1}$ ____________ **1st Result**

$\mathbf{d_{1x}, d_{2x}, d_{3x}, d_{4x}, \_\_\_\_\_\_\_\_\_\_\_\_ d_{nx}}$

$\mathbf{d_{1x}, d_{1x} . \frac{u}{V}, d_{1x}. \frac{U^2}{V^2}, d_{1x} . \frac{U^3}{V^3}, \_\_\_\_\_\_, d_{1x}. \left(\frac{U}{V}\right)^{n-1}}$

$\mathbf{d_{1x}, d_{2x} \_\_\_\_\_\_\_\_\_\_\_\_ d_{nx}}$ are also in geometric progression.

Therefore, $\mathbf{d_{nx} = d_{1x} . \left(\frac{U}{V}\right)^{n-1}}$ ____________ **2nd Result**

$$\mathbf{d_n = \sqrt{d_{nx}^{\ 2} + d_y^{\ 2}}}$$

$$\mathbf{d_n = \sqrt{\left[d_{1x} . \left(\frac{U}{V}\right)^{n-1}\right]^2 + d_y^{\ 2}}}$$ ____________ **3rd Result**

$\frac{t_{n+1}}{t_n} = \frac{d_{(a+1)x}}{d_{ax}} = \frac{u}{V}$ ____________ **4th Result**

In time 't_n' body 'A' travels distance '$d_{(a+1)x}$'and reaches to its (n +1)th position from its nth position.____________ **5th Result**

In time 't_n' body 'B' travels distance 'd_{nx}' and reaches to its (n + 1)th position from its nth position.____________ **6thResult**

When the body 'A' and 'B' are at their nth position the distance between them is $\mathbf{d_n}$

$$\mathbf{d_n = \sqrt{d_{nx}^{\ 2} + d_y^{\ 2}}}$$ ____________ **7thResult**

$\mathbf{t_1 = \frac{d_{1x}}{V}, t_2 = \frac{d_{2x}}{V}, t_3 = \frac{d_{3x}}{V}}$

$\mathbf{t_n = \frac{d_{nx}}{V}}$ ____________ **8thResult**

$\mathbf{d_{2x} = U\ t_1, d_{3x} = U\ t_2, d_{4x} = U\ t_3}$

$\mathbf{d_{(n+1)x} = U\ t_n}$ ____________ **9thResult**

The total time required by the body 'A' and body 'B' to reach at their (n +1)th position from their 1st position is denoted by '$\mathbf{t}$'.

'$\mathbf{t}$' is the sum of all the times from t_1 to t_n

$\mathbf{t = t_1 + t_2 \_\_\_\_\_\_\_\_\_\_\_\_\_ + t_n}$

We know that,

$\mathbf{S_n = \frac{a}{1-r}(1 - r^n)}$ ____________ **(A)**

$\mathbf{S_n}$ = **'t'**

$\mathbf{a = t_1 = \frac{d_{1x}}{V}}$, $\mathbf{r = \frac{U}{V}}$ substitute these values in equation (A)

$$\mathbf{t = \frac{\frac{d_{1x}}{V}}{1-\left(\frac{U}{V}\right)} \cdot \left[1 - \left(\frac{U}{V}\right)^n\right]}$$

$\mathbf{t = \frac{d_{1x}}{V-U}\left[1 - \left(\frac{U}{V}\right)^n\right]}$ ____________ **10th Result**

$\mathbf{d_A = U\ t}$

$\mathbf{d_A = \frac{d_{1x}.U}{V-U}\left[1 - \left(\frac{U}{V}\right)^n\right]}$ ____________ **11th Result**

$\mathbf{d_B = V\ t}$

$d_B = \mathbf{\frac{d_{1x}.V}{V-U}\left[1 - \left(\frac{U}{V}\right)^n\right]}$ ____________ **12th Result**

Where 'd_A' and 'd_B' are the total distances travelled by the body 'A' and body 'B' in time 't'.

3rd CASE (When the Bodies are Moving in Non Parallel Lines)

- **Kinematics of Two Bodies in Terms of Geometric Series Along X-Axis**

Suppose body 'A' and 'B' are at rest and at their 1st position the distance of body 'B' to body 'A' is $\overrightarrow{d_1} = d_1x\ \hat{i} + d_1y\ \hat{j}$ as shown in figure '5'. In the same time both the bodies start moving with uniform velocities. Body 'A' start moving with velocity $\vec{U} = U_x\hat{i} + U_y\hat{j}$ in line 'L_1' and body 'B' start moving with velocity $\vec{V} = V_x\hat{i} + V_y\hat{j}$ in line 'L_2'. Where d_{1x} ,d_{1y} ,U_x , U_y , V_x and V_y are positive. Line 'L_1' and 'L_2' are non-parallel. Body 'B' takes time 't_1' to reach at the x-co-ordinate of body 'A'. In time 't_1' body 'A' will travel distance $\overrightarrow{d_{1A}}$ and 'B' will travel the distance $\overrightarrow{d_{1B}}$. Now 'A' and 'B' are in their 2nd position and distance of 'B' to 'A' is $\overrightarrow{d_2}$ as shown in figure '6'.Then body 'B' takes time 't_2' to reach at the 2nd x co-ordinate of body 'A'. In time 't_2' body 'A' will travel distance '$\overrightarrow{d_{2A}}$' and body 'B' will travel distance '$\overrightarrow{d_{2B}}$' now both the bodies are at their 3rd position and the distance of body 'B' to 'A' is '$\overrightarrow{d_3}$' and so on. Follow figure '5' and '6'.

$\mathbf{t_1 = \frac{d_{1x}}{V_x}}$

$\mathbf{\overrightarrow{d_{1A}} = \vec{U}\ t_1}$

Where, $\mathbf{\vec{U} = (U_x\hat{i} + U_y\hat{j}), t_1 = \frac{d_{1x}}{V_x}}$

$\mathbf{\overrightarrow{d_{1A}} = \left(U_x.\frac{d_{1x}}{V_x}\right)\hat{i} + \left(U_y.\frac{d_{1x}}{V_x}\right)\hat{j}}$

$\mathbf{\overrightarrow{d_{1B}} = \vec{V}\ t_1}$

$\mathbf{\overrightarrow{d_{1B}} = (V_x\hat{i} + V_y\hat{j}).\left(\frac{d_{1x}}{V_x}\right)}$

$\mathbf{\overrightarrow{d_{1B}} = (d_{1x})\hat{i} + \left(V_y.\frac{d_{1x}}{V_x}\right)\hat{j}}$

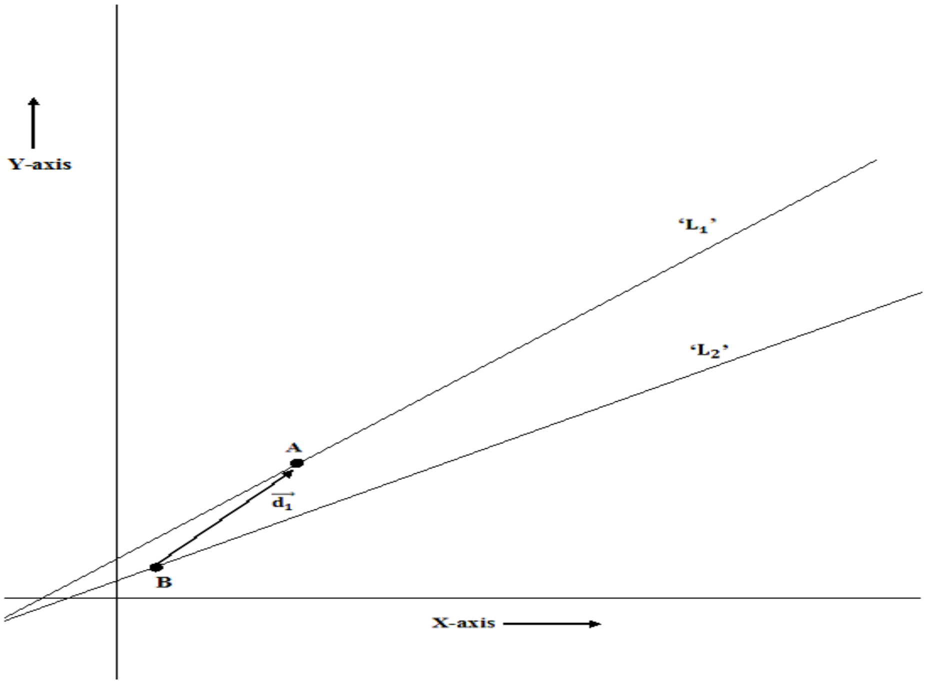

Figure 5: Initially Both the Bodies are at Rest and at Their 1st Position

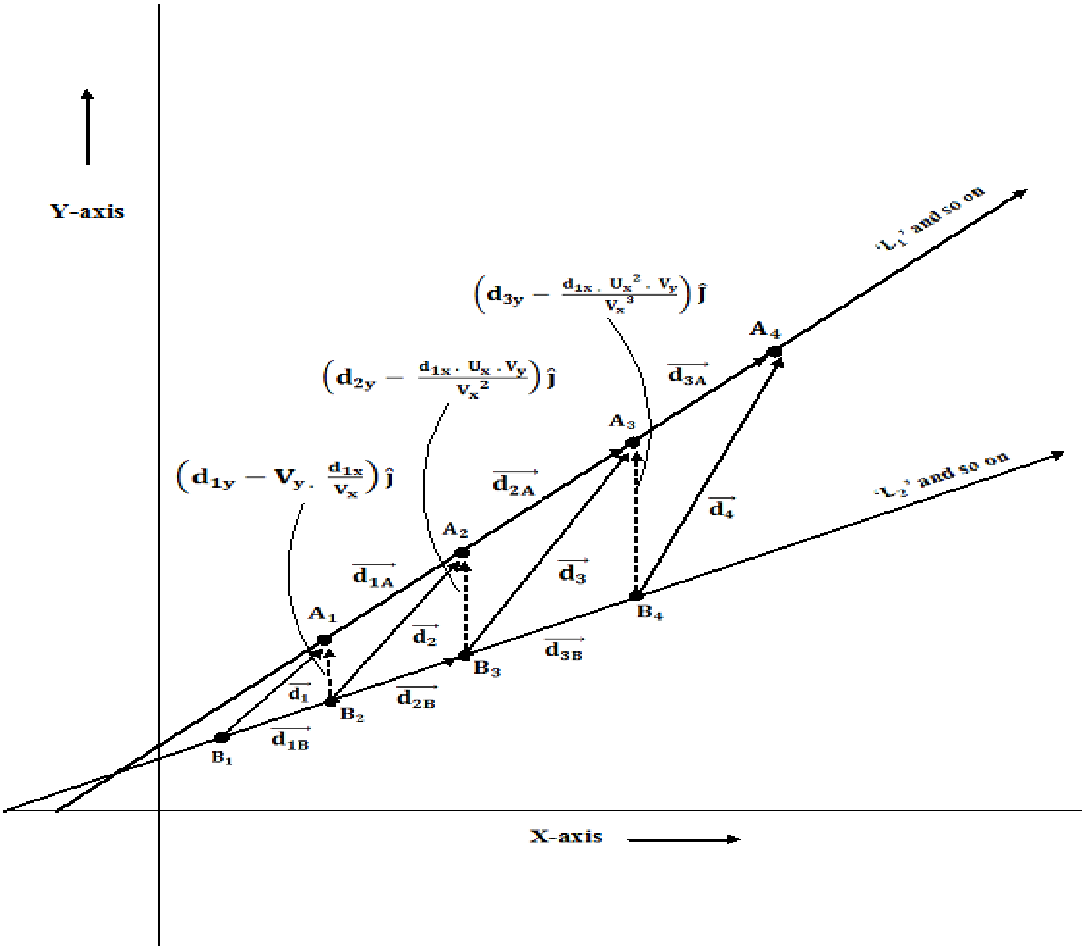

Figure 6: In Same Time Both the Bodies Start Moving with Uniform Velocities and in Non Parallel Lines

$$\overrightarrow{d_2} = \left(d_{1y} - V_y.\frac{d_{1x}}{V_x}\right)\hat{j} + \overrightarrow{d_{1A}}$$

$$\overrightarrow{\mathbf{d_2}} = \left(\mathbf{d_{1y}} - \mathbf{V_y}.\frac{\mathbf{d_{1x}}}{\mathbf{V_x}}\right)\hat{\mathbf{j}} + \left(\mathbf{U_x}.\frac{\mathbf{d_{1x}}}{\mathbf{V_x}}\right)\hat{\mathbf{i}} + \left(\mathbf{U_y}.\frac{\mathbf{d_{1x}}}{\mathbf{V_x}}\right)\hat{\mathbf{j}}$$

$$\overrightarrow{\mathbf{d_2}} = \left(\mathbf{U_x}.\frac{\mathbf{d_{1x}}}{\mathbf{V_x}}\right)\hat{\mathbf{i}} + \left(\mathbf{d_{1y}} - \mathbf{V_y}.\frac{\mathbf{d_{1x}}}{\mathbf{V_x}} + \mathbf{U_y}.\frac{\mathbf{d_{1x}}}{\mathbf{V_x}}\right)\hat{\mathbf{j}}$$

$$\mathbf{d_{2x}} = \mathbf{U_x}.\frac{\mathbf{d_{1x}}}{\mathbf{V_x}}$$

$$\mathbf{d_{2y}} = \mathbf{d_{1y}} - \mathbf{V_y}.\frac{\mathbf{d_{1x}}}{\mathbf{V_x}} + \mathbf{U_y}.\frac{\mathbf{d_{1x}}}{\mathbf{V_x}}$$

$$\mathbf{t_2} = \frac{\mathbf{d_{2x}}}{\mathbf{V_x}}$$

Where $\mathbf{d_{2x}} = \mathbf{U_x} \cdot \frac{\mathbf{d_{1x}}}{\mathbf{V_x}}$

$$\mathbf{t_2} = \frac{\mathbf{d_{1x}}}{\mathbf{V_x}} \cdot \frac{\mathbf{U_x}}{\mathbf{V_x}}$$

$$\overrightarrow{\mathbf{d_{2A}}} = \vec{\mathbf{U}}\mathbf{t_2}$$

$$\overrightarrow{\mathbf{d_{2A}}} = \left(\mathbf{U_x}\hat{\mathbf{i}} + \mathbf{U_y}\hat{\mathbf{j}}\right)\left(\frac{\mathbf{d_{1x}}}{\mathbf{V_x}} \cdot \frac{\mathbf{U_x}}{\mathbf{V_x}}\right)$$

$$\overrightarrow{\mathbf{d_{2A}}} = \left(\mathbf{d_{1x}} \cdot \frac{\mathbf{U_x}^2}{\mathbf{V_x}^2}\right)\hat{\mathbf{i}} + \left(\frac{\mathbf{d_{1x}} \cdot \mathbf{U_x} \cdot \mathbf{U_y}}{\mathbf{V_x}^2}\right)\hat{\mathbf{j}}$$

$$\overrightarrow{\mathbf{d_{2B}}} = \vec{\mathbf{V}}\mathbf{t_2}$$

$$\overrightarrow{\mathbf{d_{2B}}} = \left(\mathbf{V_x}\hat{\mathbf{i}} + \mathbf{V_y}\hat{\mathbf{j}}\right)\left(\frac{\mathbf{d_{1x}}}{\mathbf{V_x}} \cdot \frac{\mathbf{U_x}}{\mathbf{V_x}}\right)$$

$$\overrightarrow{\mathbf{d_{2B}}} = \left(\mathbf{d_{1x}} \cdot \frac{\mathbf{U_x}}{\mathbf{V_x}}\right)\hat{\mathbf{i}} + \left(\frac{\mathbf{d_{1x}} \cdot \mathbf{U_x} \cdot \mathbf{V_y}}{\mathbf{V_x}^2}\right)\hat{\mathbf{j}}$$

$$\overrightarrow{\mathbf{d_3}} = \left(\mathbf{d_{2y}} - \frac{\mathbf{d_{1x}} \cdot \mathbf{U_x} \cdot \mathbf{V_y}}{\mathbf{V_x}^2}\right)\hat{\mathbf{j}} + \overrightarrow{\mathbf{d_{2A}}}$$

Substitute the values of d_{2y} and $\overrightarrow{d_{2A}}$ in above equation.

$$\overrightarrow{\mathbf{d_3}} = \left(\mathbf{d_{1y}} - \mathbf{V_y} \cdot \frac{\mathbf{d_{1x}}}{\mathbf{V_x}} + \mathbf{U_y} \cdot \frac{\mathbf{d_{1x}}}{\mathbf{V_x}} - \frac{\mathbf{d_{1x}} \cdot \mathbf{U_x} \cdot \mathbf{V_y}}{\mathbf{V_x}^2}\right)\hat{\mathbf{j}} + \left(\mathbf{d_{1x}} \cdot \frac{\mathbf{U_x}^2}{\mathbf{V_x}^2}\right)\hat{\mathbf{i}} + \left(\frac{\mathbf{d_{1x}} \cdot \mathbf{U_x} \cdot \mathbf{U_y}}{\mathbf{V_x}^2}\right)\hat{\mathbf{j}}$$

$$\overrightarrow{\mathbf{d_3}} = \left(\mathbf{d_{1x}} \cdot \frac{\mathbf{U_x}^2}{\mathbf{V_x}^2}\right)\hat{\mathbf{i}} + \left(\mathbf{d_{1y}} - \mathbf{V_y} \cdot \frac{\mathbf{d_{1x}}}{\mathbf{V_x}} - \frac{\mathbf{d_{1x}} \cdot \mathbf{U_x} \cdot \mathbf{V_y}}{\mathbf{V_x}^2} + \mathbf{U_y}.\frac{\mathbf{d_{1x}}}{\mathbf{V_x}} + \frac{\mathbf{d_{1x}} \cdot \mathbf{U_x} \cdot \mathbf{U_y}}{\mathbf{V_x}^2}\right)\hat{\mathbf{j}}$$

$$\mathbf{d_{3x}} = \mathbf{d_{1x}} \cdot \frac{\mathbf{U_x}^2}{\mathbf{V_x}^2}$$

$$\mathbf{d_{3y}} = \mathbf{d_{1y}} - \left(\mathbf{V_y}.\frac{\mathbf{d_{1x}}}{\mathbf{V_x}} + \frac{\mathbf{d_{1x}} \cdot \mathbf{U_x} \cdot \mathbf{V_y}}{\mathbf{V_x}^2}\right) + \left(\mathbf{U_y} \cdot \frac{\mathbf{d_{1x}}}{\mathbf{V_x}} + \frac{\mathbf{d_{1x}} \cdot \mathbf{U_x} \cdot \mathbf{U_y}}{\mathbf{V_x}^2}\right)$$

$$\mathbf{t_3} = \frac{\mathbf{d_{3x}}}{\mathbf{V_x}}$$

Where $\mathbf{d_{3x}} = \mathbf{d_{1x}} \cdot \frac{\mathbf{U_x}^2}{\mathbf{V_x}^2}$

$$\mathbf{t_3} = \left(\frac{\mathbf{d_{1x}}}{\mathbf{V_x}}\right) \cdot \left(\frac{\mathbf{U_x}^2}{\mathbf{V_x}^2}\right)$$

$$\overrightarrow{\mathbf{d_{3A}}} = \vec{\mathbf{U}}\,\mathbf{t_3}$$

$$\overrightarrow{\mathbf{d_{3A}}} = (\mathbf{U_x\hat{i}} + \mathbf{U_y\hat{j}})\left(\frac{\mathbf{d_{1x}}}{\mathbf{V_x}} \cdot \frac{\mathbf{U_x}^2}{\mathbf{V_x}^2}\right)$$

$$\overrightarrow{\mathbf{d_{3A}}} = \left(\mathbf{d_{1x}} \cdot \frac{\mathbf{U_x}^3}{\mathbf{V_x}^3}\right)\hat{\mathbf{i}} + \left(\frac{\mathbf{d_{1x}} \cdot \mathbf{U_y} \cdot \mathbf{U_x}^2}{\mathbf{V_x}^3}\right)\hat{\mathbf{j}}$$

$$\overrightarrow{\mathbf{d_{3B}}} = \vec{\mathbf{V}}\,\mathbf{t_3}$$

$$\overrightarrow{\mathbf{d_{3B}}} = (\mathbf{V_x\hat{i}} + \mathbf{V_y\hat{j}})\left(\frac{\mathbf{d_{1x}}}{\mathbf{V_x}} \cdot \frac{\mathbf{U_x}^2}{\mathbf{V_x}^2}\right)$$

$$\overrightarrow{\mathbf{d_{3B}}} = \left(\mathbf{d_{1x}} \cdot \frac{\mathbf{U_x}^2}{\mathbf{V_x}^2}\right)\hat{\mathbf{i}} + \left(\frac{\mathbf{d_{1x}} \cdot \mathbf{U_x}^2 \cdot \mathbf{V_y}}{\mathbf{V_x}^3}\right)\hat{\mathbf{j}}$$

$$\overrightarrow{\mathbf{d_4}} = \left(\mathbf{d_{3y}} - \frac{\mathbf{d_{1x}} \cdot \mathbf{U_x}^2 \cdot \mathbf{V_y}}{\mathbf{V_x}^3}\right)\hat{\mathbf{j}} + \overrightarrow{\mathbf{d_{3A}}}$$

Substitute the values of d_{3y} and $\overrightarrow{d_{3A}}$ in above equation we get

$$\overrightarrow{\mathbf{d_4}} = \left(\mathbf{d_{1x}} \cdot \frac{\mathbf{U_x}^3}{\mathbf{V_x}^3}\right)\hat{\mathbf{i}} + \left(\mathbf{d_{1y}} - \mathbf{V_y} \cdot \frac{\mathbf{d_{1x}}}{\mathbf{V_x}} - \frac{\mathbf{d_{1x}} \cdot \mathbf{U_x} \cdot \mathbf{V_y}}{\mathbf{V_x}^2} - \frac{\mathbf{d_{1x}} \cdot \mathbf{U_x}^2 \cdot \mathbf{V_y}}{\mathbf{V_x}^3} + \mathbf{U_y} \cdot \frac{\mathbf{d_{1x}}}{\mathbf{V_x}} + \frac{\mathbf{d_{1x}} \cdot \mathbf{U_x} \cdot \mathbf{U_y}}{\mathbf{V_x}^2} + \frac{\mathbf{d_{1x}} \cdot \mathbf{U_x}^2 \cdot \mathbf{U_y}}{\mathbf{V_x}^3}\right)\hat{\mathbf{j}}$$

$$\overrightarrow{\mathbf{d_{1A}}} = \vec{\mathbf{U}}\,\mathbf{t_1}$$

$$\overrightarrow{\mathbf{d_{2A}}} = \vec{\mathbf{U}}\,\mathbf{t_2}$$

$$\overrightarrow{\mathbf{d_{3A}}} = \vec{\mathbf{U}}\,\mathbf{t_3}$$

Therefore, $\overrightarrow{\mathbf{d_{nA}}} = \vec{\mathbf{U}}\,\mathbf{t_n}$__________ **1st Result.**

Also,

$$\overrightarrow{\mathbf{d_{1B}}} = \vec{\mathbf{V}}\,\mathbf{t_1}$$

$$\overrightarrow{\mathbf{d_{2B}}} = \vec{\mathbf{V}}\,\mathbf{t_2}$$

$$\overrightarrow{\mathbf{d_{3B}}} = \vec{\mathbf{V}}\,\mathbf{t_3}$$

Therefore, $\overrightarrow{\mathbf{d_{nB}}} = \vec{\mathbf{V}}\,\mathbf{t_n}$__________ **2ndResult.**

$$\mathbf{t_1} = \frac{\mathbf{d_{1x}}}{\mathbf{V_x}}$$

$$\mathbf{t_2} = \frac{\mathbf{U_x}}{\mathbf{V_x}} \cdot \frac{\mathbf{d_{1x}}}{\mathbf{V_x}}$$

$$\mathbf{t_3} = \frac{\mathbf{U_x}^2}{\mathbf{V_x}^2} \cdot \frac{\mathbf{d_{1x}}}{\mathbf{V_x}}$$

$$\frac{\mathbf{t_2}}{\mathbf{t_1}} = \frac{\mathbf{t_3}}{\mathbf{t_2}} = \frac{\mathbf{t_{n+1}}}{\mathbf{t_n}} = \frac{\mathbf{U_x}}{\mathbf{V_x}}$$

Therefore, $\mathbf{t_n} = \frac{\mathbf{d_{1x}}}{\mathbf{V_x}} \cdot \left(\frac{\mathbf{U_x}}{\mathbf{V_x}}\right)^{\mathbf{n-1}}$__________ **3rd Result.**

Put $\mathbf{t_n} = \frac{\mathbf{d_{1x}}}{\mathbf{V_x}} \cdot \left(\frac{\mathbf{U_x}}{\mathbf{V_x}}\right)^{\mathbf{n-1}}$ in 1st result.

$$\overrightarrow{\mathbf{d_{nA}}} = \vec{\mathbf{U}}\,\mathbf{t_n}$$

Where $\vec{\mathbf{U}} = (\mathbf{U_x}\hat{\mathbf{i}} + \mathbf{U_y}\hat{\mathbf{j}})$

$\overrightarrow{\mathbf{d_{nA}}} = \left[\mathbf{U}_x.\frac{\mathbf{d_{1x}}}{\mathbf{V_x}}.\left(\frac{\mathbf{U_x}}{\mathbf{V_x}}\right)^{\mathbf{n-1}}\right]\hat{\mathbf{i}} + \left[\mathbf{U}_y.\frac{\mathbf{d_{1x}}}{\mathbf{V_x}}.\left(\frac{\mathbf{U_x}}{\mathbf{V_x}}\right)^{\mathbf{n-1}}\right]\hat{\mathbf{j}}$

$\overrightarrow{\mathbf{d_{nA}}} = \left[\mathbf{d_{1x}}.\left(\frac{\mathbf{U_x}}{\mathbf{V_x}}\right)^{\mathbf{n}}\right]\hat{\mathbf{i}} + \left[\mathbf{U}_y.\frac{\mathbf{d_{1x}}}{\mathbf{V_x}}.\left(\frac{\mathbf{U_x}}{\mathbf{V_x}}\right)^{\mathbf{n-1}}\right]\hat{\mathbf{j}}$ _________ **4th Result.**

Put $\mathbf{t_n} = \frac{\mathbf{d_{1x}}}{\mathbf{V_x}}.\left(\frac{\mathbf{U_x}}{\mathbf{V_x}}\right)^{\mathbf{n-1}}$ in 2nd Result

$\overrightarrow{\mathbf{d_{nB}}} = (\mathbf{V_x}\hat{\mathbf{i}} + \mathbf{V_y}\hat{\mathbf{j}})\left[\frac{\mathbf{d_{1x}}}{\mathbf{V_x}}.\left(\frac{\mathbf{U_x}}{\mathbf{V_x}}\right)^{\mathbf{n-1}}\right]$

$\overrightarrow{\mathbf{d_{nB}}} = \left[\mathbf{d_{1x}}.\left(\frac{\mathbf{U_x}}{\mathbf{V_x}}\right)^{\mathbf{n-1}}\right]\hat{\mathbf{i}} + \left[\frac{\mathbf{d_{1x}}.\mathbf{V_y}}{\mathbf{V_x}}\left(\frac{\mathbf{U_x}}{\mathbf{V_x}}\right)^{\mathbf{n-1}}\right]\hat{\mathbf{j}}$_________ **5th Result.**

We know that

$\overrightarrow{\mathbf{d_1}} = \mathbf{d_{1x}}\hat{\mathbf{i}} + \mathbf{d_{1y}}\hat{\mathbf{j}}$

$\overrightarrow{\mathbf{d_2}} = \left(\mathbf{d_{1x}}.\frac{\mathbf{U_x}}{\mathbf{V_x}}\right)\hat{\mathbf{i}} + \left(\mathbf{d_{1y}} - \mathbf{V_y}.\frac{\mathbf{d_{1x}}}{\mathbf{V_x}} + \mathbf{U_y}.\frac{\mathbf{d_{1x}}}{\mathbf{V_x}}\right)\hat{\mathbf{j}}$

$\overrightarrow{\mathbf{d_3}} = \left(\mathbf{d_{1x}}.\frac{\mathbf{U_x}^2}{\mathbf{V_x}^2}\right)\hat{\mathbf{i}} + \left[\mathbf{d_{1y}} - \left(\mathbf{V_y}.\frac{\mathbf{d_{1x}}}{\mathbf{V_x}} + \frac{\mathbf{V_y}.\mathbf{U_x}.\mathbf{d_{1x}}}{\mathbf{V_x}^2}\right) + \left(\mathbf{U_y}.\frac{\mathbf{d_{1x}}}{\mathbf{V_x}} + \frac{\mathbf{U_y}.\mathbf{U_x}.\mathbf{d_{1x}}}{\mathbf{V_x}^2}\right)\right]\hat{\mathbf{j}}$

$\overrightarrow{\mathbf{d_4}} = \left(\mathbf{d_{1x}}.\frac{\mathbf{U_x}^3}{\mathbf{V_x}^3}\right)\hat{\mathbf{i}} + \left[\mathbf{d_{1y}} - \left(\mathbf{V_y}.\frac{\mathbf{d_{1x}}}{\mathbf{V_x}} + \frac{\mathbf{V_y}.\mathbf{U_x}.\mathbf{d_{1x}}}{\mathbf{V_x}^2} + \frac{\mathbf{V_y}.\mathbf{U_x}^2.\mathbf{d_{1x}}}{\mathbf{V_x}^3}\right) + \left(\mathbf{U_y}.\frac{\mathbf{d_1x}}{\mathbf{V_x}} + \frac{\mathbf{U_y}.\mathbf{U_x}.\mathbf{d_{1x}}}{\mathbf{V_x}^2} + \frac{\mathbf{U_y}.\mathbf{U_x}^2.\mathbf{d_{1x}}}{\mathbf{V_x}^3}\right)\right]\hat{\mathbf{j}}$

Now I am finding $\overrightarrow{\mathrm{d_n}}$

$\mathbf{V_y}.\frac{\mathbf{d_{1x}}}{\mathbf{V_x}}, \frac{\mathbf{V_y}.\mathbf{d_{1x}}.\mathbf{U_x}}{\mathbf{V_x}^2}, \frac{\mathbf{V_y}.\mathbf{d_{1x}}.\mathbf{U_x}^2}{\mathbf{V_x}^3}$ ------------------------- are in geometric progression in which

$\frac{\mathbf{t_2}}{\mathbf{t_1}} = \frac{\mathbf{t_3}}{\mathbf{t_2}} = \frac{\mathbf{U_x}}{\mathbf{V_x}} = \mathbf{r}, \mathbf{a} = \mathbf{V_y}.\frac{\mathbf{d_{1x}}}{\mathbf{V_x}}$

We know that $\mathbf{S_n} = \frac{\mathbf{a}}{\mathbf{1-r}}(\mathbf{1} - \mathbf{r^n})$

$\mathbf{S_{n-1}} = \frac{\mathbf{a}}{\mathbf{1-r}}(\mathbf{1} - \mathbf{r^{n-1}})$

Put $\mathbf{a} = \mathbf{V_y}.\frac{\mathbf{d_{1x}}}{\mathbf{V_x}}$ and $\mathbf{r} = \frac{\mathbf{U_x}}{\mathbf{V_x}}$

$\mathbf{S_{n-1}} = \frac{\mathbf{V}_y.\frac{\mathbf{d_{1x}}}{\mathbf{V_x}}}{1-\left(\frac{\mathbf{U_x}}{\mathbf{V_x}}\right)}\left[\mathbf{1} - \left(\frac{\mathbf{U_x}}{\mathbf{V_x}}\right)^{\mathbf{n-1}}\right]$

$\mathbf{S_{n-1}} = \frac{\mathbf{V_y}.\mathbf{d_{1x}}}{\mathbf{V_x} - \mathbf{U_x}}\left[\mathbf{1} - \left(\frac{\mathbf{U_x}}{\mathbf{V_x}}\right)^{\mathbf{n-1}}\right]$

Also,

$\mathbf{U_y}.\frac{\mathbf{d_{1x}}}{\mathbf{V_x}}, \frac{\mathbf{U_y}.\mathbf{d_{1x}}.\mathbf{U_x}}{\mathbf{V_x}^2}, \frac{\mathbf{U_y}.\mathbf{d_{1x}}.\mathbf{U_x}^2}{\mathbf{V_x}^3}$, --------------------------- are in geometric progression in which $\mathbf{a} = \mathbf{U_y}.\frac{\mathbf{d_{1x}}}{\mathbf{V_x}}, \mathbf{r} = \frac{\mathbf{U_x}}{\mathbf{V_x}}$

We know that $\mathbf{S_n} = \frac{\mathbf{a}}{\mathbf{1-r}}(\mathbf{1} - \mathbf{r^n})$

$\boldsymbol{S_{n-1}} = \frac{\mathbf{a}}{\mathbf{1-r}}(\mathbf{1} - \mathbf{r^{n-1}})$

Put $\mathbf{a} = \mathbf{U_y} \cdot \frac{\mathbf{d_{1x}}}{\mathbf{V_x}}$, $\mathbf{r} = \frac{\mathbf{U_x}}{\mathbf{V_x}}$ in above equation

$$S_{n-1} = \frac{U_y \frac{d_{1x}}{V_x}}{1-\left(\frac{U_x}{V_x}\right)}\left[1 - \left(\frac{U_x}{V_x}\right)^{n-1}\right]$$

$$S_{n-1} = \frac{U_y \cdot d_{1x}}{V_x - U_x}\left[1 - \left(\frac{U_x}{V_x}\right)^{n-1}\right]$$

$$\overrightarrow{d_1} = d_{1x}\,\hat{\imath} + d_{1y}\,\hat{\jmath}$$

$$\overrightarrow{d_2} = \left(d_{1x} \cdot \frac{U_x}{V_x}\right)\hat{\imath} + \left(d_{1y} - V_y \cdot \frac{d_{1x}}{V_x} + U_y \cdot \frac{d_{1x}}{V_x}\right)\hat{\jmath}$$

$$\overrightarrow{d_n} = \left[d_{1x} \cdot \left(\frac{U_x}{V_x}\right)^{n-1}\right]\hat{\imath} + \left[d_{1y} - \left\{\frac{V_y \cdot d_{1x}}{V_x - U_x}\left(1 - \left(\frac{U_x}{V_x}\right)^{n-1}\right)\right\} + \left\{\frac{U_y \cdot d_{1x}}{V_x - U_x}\left(1 - \left(\frac{U_x}{V_x}\right)^{n-1}\right)\right\}\right]\hat{\jmath}$$ --- **6th Result**

Body 'A' takes time 't_n' and travel distance $\overrightarrow{d_{nA}}$ to reach at its $(n+1)^{th}$ position from its n^{th} position. ____________ **7th Result.**

Body 'B' takes time 't_n' and travel distance '$\overrightarrow{d_{nB}}$' to reach at its $(n+1)^{th}$ position from its n^{th} position. ____________ **8th Result.**

When the body 'A' and body 'B' are at their n^{th} position the distance of body 'B' to body 'A' is $\overrightarrow{d_n}$. ____________ **9th Result.**

The total time required by the body 'A' and body 'B' to reach at their $(n+1)^{th}$ position from their 1^{st} position is **'t'.** Also **t** is the sum of all the times from 't_1' to 't_n'.

$$t = t_1 + t_2 + t_3 \text{ ________________ } + t_n$$

We know that $\mathbf{S_n} = \frac{a}{1-r}(1 - r^n)$

$\mathbf{S_n} = \mathbf{t}$, $\mathbf{a} = \mathbf{t_1} = \frac{d_{1x}}{V_x}$, $\mathbf{r} = \frac{U_x}{V_x}$

Substitute all these values in above equation

$$t = \frac{\frac{d_{1x}}{V_x}}{1-\left(\frac{U_x}{V_x}\right)}\left[1 - \left(\frac{U_x}{V_x}\right)^n\right]$$

$$t = \frac{d_{1x}}{V_x - U_x}\left[1 - \left(\frac{U_x}{V_x}\right)^n\right]$$ ____________ **10th Result.**

The total distance travelled by the body 'A' in time '**t**' is $\overrightarrow{\mathbf{D_A}}$.

$$\overrightarrow{D_A} = \vec{U}\,t$$

$$\overrightarrow{D_A} = \left(U_x\hat{\imath} + U_y\hat{\jmath}\right)\left[\frac{d_{1x}}{V_x - U_x}\left(1 - \left(\frac{U_x}{V_x}\right)^n\right)\right]$$

$$\overrightarrow{D_A} = \left[\frac{d_{1x} \cdot U_x}{V_x - U_x}\left\{1 - \left(\frac{U_x}{V_x}\right)^n\right\}\right]\hat{\imath} + \left[\frac{d_{1x} \cdot U_y}{V_x - U_x}\left\{1 - \left(\frac{U_x}{V_x}\right)^n\right\}\right]\hat{\jmath}$$ ________ **11th Result.**

The total distance travelled by the body 'B' in time **'t'** is $\overrightarrow{\mathbf{D_B}}$**.**

$$\overrightarrow{D_B} = \vec{V}\,t$$

$$\overrightarrow{D_B} = \left(V_x\,\hat{i} + V_y\,\hat{j}\right)\left[\frac{d_{1x}}{V_x - U_x}\left(1 - \left(\frac{U_x}{V_x}\right)^n\right)\right]$$

$$\overrightarrow{D_B} = \left[\frac{d_{1x}\,.V_x}{V_x - U_x}\left\{1 - \left(\frac{U_x}{V_x}\right)^n\right\}\right]\hat{i} + \left[\frac{d_{1x}\,V_y}{V_x - U_x}\left\{1 - \left(\frac{U_x}{V_x}\right)^n\right\}\right]\hat{j}$$ ________ **12^{th} Result.**

CONCLUSIONS

It is obvious that motion of two bodies with uniform velocity can be described in terms of Geometric Series. A tool is proved "Geometric Progression" for the easiness of system of two bodies moving in straight line with uniform velocities.

ADVANTAGES OF USING THIS RESEARCH

- This research is very important in point to point analysis for system of bodies moving with uniform velocities.
- By using this research we can find the exact position of one body with respect to another body at any instant of time.
- We know that when two bodies are moving with uniform velocities then their time and distance varies according to geometric progression hence we can easily apply all the formulas of geometric progression in it.
- This research will give 100% accurate results.
- This research can help scientists for the point to point analysis of the bodies which are moving in any curve path for example circular path, elliptical path etc.
- This research can help scientists to find the position of other moving objects in space with respect to earth at any instant of time.
- By using this research we can easily perform complex and lengthy calculations by putting the values in equations for example. 1) If we have to find the distance between two bodies when they are at their 1000000000^{th} position then we can easily calculate by simply putting n= 10^9. 2) If we have to find the time required by both the bodies to reach at their 10000^{th} position from their 9999^{th} position then we can easily calculate by putting n= 9999 in the equation of time as derived in results and discussions. 3) Many other parameters can be easily calculated by using this research.
- By using this research we can easily find the direction of distance and velocities of objects moving with uniform velocities.

BACKGROUND

Once I had watched a movie in which a group of boys was having a racing competition between them. When the race started a boy fell down due to which he was left behind. I got the idea that in order to win the race the velocity of the boy left behind should be greater than the velocity of boy at front. Then I assumed that the velocity of the boy at back is **V** and the velocity of boy at front is **U** and the distance between is $\mathbf{d_1}$. I used the approach that when the boy at back reaches to the position of boy at front then in the same time the boy at front travels the distance $\mathbf{d_2}$. When the boy at back reaches the 2^{nd} position of the boy at front then in the same time the boy at front travels the distance $\mathbf{d_3}$. Then I found that $\mathbf{d_1}$, $\mathbf{d_2}$, $\mathbf{d_3}$, __________, $\mathbf{d_n}$ and $\mathbf{t_1}$, $\mathbf{t_2}$, $\mathbf{t_3}$, ___________ $\mathbf{t_n}$ are in geometric progression. Then by using the formulas of geometric series I derived the equations.

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