International Journal of Mechanical Engineering (IJME)
ISSN(P): 2319-2240; ISSN(E): 2319-2259
Vol. 3, Issue 1, Jan 2014, 73- 82

International Academy of Science, Engineering and Technology
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EFFECT OF DUSTY FLUID ON MHD FREE CONVECTION FLOW THROUGH POROUS MEDIUM BY AN OSCILLATING POROUS PLATE IN PRESENCE OF CHEMICAL REACTION, HEAT SOURCE AND CONSTANT SUCTION

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ABSTRACT

Effects of dusty fluid on MHD free convection flow through porous medium by an oscillating porous plate in presence of chemical reaction, heat source and constant suction is investigated. The dimensionless governing equations are solved using oscillatory flow conditions. The results are obtained for velocity profile for both components, heat profile, mass concentration profile and shearing stress for different parameters like Schmidt number, time, magnetic parameter, porous parameter, dusty parameter etc. The flow characteristics are discussed and shown by means of graphs and table.

KEYWORDS: Heat Transfer, Mass Transfer, Oscillatory Flow, MHD, Chemical Reaction, Heat Source and Porous Medium

INTRODUCTION

Combined heat and mass transfer problems are important in many processes and have therefore received a considerable amount of attention. In many mass transfer processes, heat transfer considerations arise due to chemical reaction and often due to the very nature of the process. In processes, such as drying, evaporation at the surface of a water body, energy transfer in a wet cooling tower and the flow in a desert cooler, heat and mass transfer occur simultaneously. Gireesh Kumar et al. [11] analyzed the effects of the chemical reaction and mass transfer on MHD unsteady free convection flow past an infinite vertical plate with constant suction and heat sink. Sattar [8] reported the free convection and mass transfer flow through a porous medium past an infinite vertical porous plate with time dependant temperature and concentration. Kim et al [7] and Harris et al [9] have studied the problem of natural convection flow through porous medium past vertical plate. Mishra and Mohapatra [1] have considered the unsteady MHD free convection flow past a vertical porous plate. Raptis et al [5] and Geindreau et al [10] studied the effect of magnetic field in flow through porous medium. Raptis, Tzivonidis and Kafousias [3] and Raptis, Kafousias and Massalas [4] have studied the steady free convection and mass transfer through porous medium. Mohapatra and Senapati [6] have considered the steady MHD free convection flow through a porous medium with mass transfer.

Datta and Jana [2] have investigated the problem of flow and heat transfer in an elastic-viscous liquid over an oscillating plate in a rotating flame.

It is proposed to study the effect of dusty fluid on MHD free convection flow through porous medium by an oscillating porous plate in presence of chemical reaction, heat source and constant suction

FORMULATION OF PROBLEM

Let us consider a free convection flow of dusty fluid which is incompressible electrically conducting through porous medium bounded by oscillating plate in presence of heat source and chemically reacting species. A uniform magnetic field B_0 is acting normal to the plate and along y'axis. T_∞' and C_∞' are temperature and mass concentration of the fluid in free stream respectively. T_w' and C_w' are respectively temperature and mass concentration at the plate. The pressure P of the fluid is assumed to be constant. All the variables are the function of y'andt' only. $V'$is the velocity of dusty fluid having mass m.Then by usual Boussinesq's approximation the unsteady flow is governed by the following equations.

$$\frac{\partial v'}{\partial y'} = 0 \Rightarrow v' = -V_0{}' \text{as the suction velocity} \tag{1}$$

$$\frac{\partial u'}{\partial t'} - V_0 \frac{\partial u'}{\partial y'} = g\beta(T' - T_\infty') + g\beta_c(C' - C_\infty') + \nu \frac{\partial^2 u'}{\partial y'^2} - \frac{\sigma B_0^2}{\rho} u'$$

$$-\frac{\nu}{K'} u' + \frac{K_0 N}{\rho}(v' - u') \tag{2}$$

$$m \frac{\partial v'}{\partial y'} = K_0(u' - v') \tag{3}$$

$$\frac{\partial T'}{\partial t'} - V_0{}' \frac{\partial T'}{\partial y'} = \alpha \frac{\partial^2 T'}{\partial y'^2} + S'(T' - T_\infty') \tag{4}$$

$$\frac{\partial C'}{\partial t'} - V_0{}' \frac{\partial C'}{\partial y'} = D \frac{\partial^2 C'}{\partial y'^2} - R'(C' - C_\infty') \tag{5}$$

With the following boundary conditions

$$\left.\begin{aligned} & u' = U_0 e^{i\omega' t'}, T' = T_w', C' = C_w' \; at \; y' = 0 \\ & u' \to 0, T' \to T_\infty', C' \to C_\infty' \; as \; y' \to \infty \end{aligned}\right\} \tag{6}$$

Where β is volumetric coefficient of thermal expansion, β_c is volumetric coefficient of thermal expansion with concentration, ν is the kinematic viscosity, σ is theelectric conductivity, K' is the permeability of porous medium, K_o is the stoke's resistance co-efficient, N is the number density of dust particles is source/sink co-efficient, D is mass diffusion and R' is chemical reaction parameter.

Let us introduce the following non-dimensional quantities

$$\left.\begin{aligned} & y = U_0 \frac{y'}{\nu}, u = \frac{u'}{U_0}, \theta = \frac{T' - T_\infty'}{T_w' - T_\infty'}, C = \frac{C' - C_\infty'}{C_w' - C_\infty'}, t = \frac{U_0^2 t'}{\nu}, V_0 = \frac{V_0'}{U_0}, \\ & \omega = \frac{\omega' \nu}{U_0^2}, v = \frac{v'}{U_0}, Gr = \frac{g\beta\nu(T' - T_\infty')}{U_0^3}, Gm = \frac{g\beta_c \nu(C' - C_\infty')}{U_0^3}, B_1 = \frac{\nu K_0 N}{\rho U_0^2}, \\ & B_2 = \frac{m U_0^2}{\nu K_0}, Pr = \frac{\nu}{\alpha}, Q = \frac{Q'\nu}{U_0^2}, Sc = \frac{\nu}{D}, K = \frac{K' U_0^2}{\nu^2}, R = \frac{\nu R'}{U_0}, M = \frac{B_0}{U_0}\sqrt{\left(\frac{\nu\sigma}{\rho}\right)} \end{aligned}\right\} \tag{7}$$

Where Gr is Grash of number, Gm modified Grash of number, M is magnetic number, Pr is prandtl number, Sc is Schmidt number, K permeability parameter porous medium ratio R is chemical reaction parameter, Q is source parameter, B_1 is dusty fluid parameter, B_2 is dust particles parameter and V_0 is the suction parameter.

The equations (2)-(5) with boundary condition (6) reduce to

$$\frac{\partial u}{\partial t} - V_0 \frac{\partial u}{\partial y} = Gr\theta + GmC + \frac{\partial^2 u}{\partial y^2} - \left(M^2 + \frac{1}{K}\right) u + B_1(v - u) \tag{8}$$

$$B_2 \frac{\partial v}{\partial t} = u - v \tag{9}$$

$$\frac{\partial \theta}{\partial t} - V_0 \frac{\partial \theta}{\partial y} = \frac{1}{Pr} \frac{\partial^2 \theta}{\partial y^2} + Q\theta \tag{10}$$

$$\frac{\partial C}{\partial t} - V_0 \frac{\partial C}{\partial y} = \frac{1}{Sc}\frac{\partial^2 C}{\partial y^2} - RC \tag{11}$$

With the boundary conditions

$$\left.\begin{array}{l} u = e^{i\omega t}, \theta = 1, C = 1 \ at\ y = 0 \\ u \to 0, \theta \to 0, C \to 0 \ as\ y \to \infty \end{array}\right\} \tag{12}$$

METHOD OF SOLUTION

To find the solution let us consider the following

$$\left.\begin{array}{l} u = u_0 + u_1 e^{i\omega t} \\ v = v_0 + v_1 e^{i\omega t} \\ \theta = \theta_0 + \theta_1 e^{i\omega t} \end{array}\right\} \tag{13}$$

$$C = C_0 + C_1 e^{i\omega t}$$

And substitute in equations (8) to (12) then by comparing both side, we get

$$\frac{d^2u_0}{dy^2} + V_0\frac{du_0}{dy} - \left(M^2 + \frac{1}{K}\right)u_0 = -(Gr\theta_0 + GmC_0) \tag{14}$$

$$\frac{d^2u_1}{dy^2} + V_0\frac{du_1}{dy} - \left(M^2 + \frac{1}{K} + i\omega + B_1 - \frac{B_1}{1+i\omega B_2}\right)u_1 = -(Gr\theta_1 + GmC_1) \tag{15}$$

$$\frac{d^2\theta_0}{dy^2} + PrV_0\frac{d\theta_0}{dy} + PrQ\theta_0 = 0 \tag{16}$$

$$\frac{d^2\theta_1}{dy^2} + PrV_0\frac{d\theta_1}{dy} + pr(Q - i\omega)\theta_1 = 0 \tag{17}$$

$$\frac{d^2C_0}{dy^2} + ScV_0\frac{dC_0}{dy} - ScRC_0 = 0 \tag{18}$$

$$\frac{d^2C_1}{dy^2} + ScV_0\frac{dC_1}{dy} - Sc(R + i\omega)C_1 = 0 \tag{19}$$

$$v_0 = u_0 \tag{20}$$

$$v_1 = \frac{u_1}{1+i\omega B_2} \tag{21}$$

With the following boundary conditions

$$\left.\begin{array}{l} u_0 = 0, u_1 = 1, \theta_0 = 1, \theta_1 = 0, C_0 = 1, C_1 = 0 \ at\ y = 0 \\ u_0 \to 0, u_1 \to 0, \theta_0 \to 0, \theta_1 \to 0, C_0 \to 0, C_1 \to 0 \ as\ y = \infty \end{array}\right\} \tag{22}$$

By solving equations (14) – (21) using the boundary conditions(22), we get

$$u = (A_7e^{-A_5y} + A_8e^{-A_1y} + A_9e^{-A_2y}) + (e^{-A_6y})e^{i\omega t} \tag{23}$$

$$v = (A_7e^{-A_5y} + A_8e^{-A_1y} + A_9e^{-A_2y}) + \left(\frac{e^{-A_6y}}{1+i\omega B_2}\right)e^{i\omega t} \tag{24}$$

$$\theta = e^{-A_1y} \tag{25}$$

$$C = e^{-A_2y} \tag{26}$$

The non-dimensional Shearing stress at the wall,

$$\tau_0 = \left(\frac{\partial u}{\partial y}\right)_{y=0} = -[(A_7A_5 + A_8A_1 + A_9A_2) + (A_6)e^{i\omega t}] \tag{27}$$

The non-dimensional rate of heat transfer, Nussle Number

$$Nu = -\left(\frac{\partial \theta}{\partial y}\right)_{y=0} = A_1 \tag{28}$$

The non-dimensional rate of mass transfer, Sherwood Number

$$Sh = -\left(\frac{\partial C}{\partial y}\right)_{y=0} = A_2 \tag{29}$$

Where $A_1 = \frac{PrV_0+\sqrt{Pr^2V_0^2-4PrQ}}{2}$, $A_2 = \frac{ScV_0+\sqrt{Sc^2V_0^2+4ScR}}{2}$

$A_8 = \frac{-Gr}{A_1^2-V_0A_1-\left(M^2+\frac{1}{K}\right)}$, $A_9 = \frac{-Gm}{A_2^2-V_0A_2-\left(M^2+\frac{1}{K}\right)}$

$S = \left(M^2 + \frac{1}{K} + i\omega + B_1 - \frac{B_1}{1+i\omega B_2}\right)$, $A_7 = A_{11} + A_{12}$

$$A_5 = \frac{-\left(V_0 + \sqrt{V_0^2 + 4\left(M^2 + \frac{1}{K}\right)}\right)}{2}, A_6 = \frac{-\left(V_0 + \sqrt{V_0^2 + 4S}\right)}{2}.$$

RESULTS AND DISCUSSIONS

In this paper we have studied the effect of dusty fluid on MHD free convection flow through porous medium by an oscillating porous plate in presence of chemical reaction, heat source and constant suction. The effect of the parameters Gr, Gm, Q, Vo, M, R, K, B_1,B_2, Pr and Sc on flow characteristics have been studied and shown by means of graphs and tables. In order to have physical correlations, we choose suitable values of flow parameters. The graphs of velocities, heat and mass concentration are taken w.r.t y. Shearing Stress is obtained in tables for different parameters.

Velocity Profiles (u): The velocity profiles are depicted in Figure 1, 3. Figure 1 shows the effect of the parameters B_1, B_2, Pr and Q on velocity at any point of the fluid, when Vo=2, Sc=0.23, Gr=2, Gm=2, R=2, M=2, K=2, t=0.01, $\omega = 0.2$. It is noticed that the velocity decreases with the increase of dusty fluid parameter (B_1), dust particles parameter (B_2), Prandtl number (Pr) and source parameter (Q)

Figure 2 shows the effect of the parameters M,R and K on velocity at any point of the fluid, when $B_1 = 1, B_2 = 1$ $Pr = 0.23$, Q=2 Vo=2, Sc=0.23, Gr=2, Gm=2, t=0.01, $\omega = 0.2$. It is noticed that the velocity decreases with the increase of Magnetic parameter (M) and chemical reaction parameter (R), where as increases with the increase of permeability parameter porous medium (K).

Figure 3 shows the effect of the parameters Gr, Gm, Sc and Vo on velocity at any point of the fluid, when $B_1 = 1, B_2 = 1, Pr = 0.23$, Q=2, K=0.23, M=2, R=2, t=0.01, $\omega = 0.2$. It is noticed that the velocity decreases with the increase of Schmidt number (Sc) and suction parameter (Vo) where as increases with the increase of Grash off number (Gr) and modified Grashoff number (Gm).

Velocity Profiles (v): The velocity profiles (v) are depicted in Figures 4, 7. Figure 4 shows the effect of the parameters Pr, Sc and Vo on velocity at any point of the fluid, when $B_1 = 1, B_2 = 1Gr = 2, Gm = 2$, Q=2, K=2, M=2, R=2, t=0.01, $\omega = 0.2$.It is noticed that the velocity decreases with the increase of suction parameter (Vo), Schmidt number (Sc) and Prandtl number (Pr)

Figure 5 shows the effect of the parameters M,Q and K on velocity at any point of the fluid**,** when $B_1 = 1, B_2 = 1$ $Gr = 2, Gm = 2,$ Sc=0.23,Pr=0.23,R=2, t=0.01,$\omega = 0.2$. It is noticed that the velocity decreases with the increase of Magnetic parameter (M) and Source parameter (Q), where as increases with the increase of permeability parameter porous medium (K).

Figure 6 shows the effect of the parameters Gm, Gr and R on velocity at any point of the fluid, when $B_1 = 1, B_2 = 1, K = 2, M = 2,$ Sc=0.23,Pr=0.23,Q=2, t=0.01,$\omega = 0.2$. It is noticed that the velocity decreases with the increase of chemical reaction parameter (R) where as increases with the increase of Grash off number (Gr) and modified Grash off number (Gm).

Figure 7 shows the effect of the parameters B_1 and B_2 on velocity at any point of the fluid, when M=2, K=2, R=2, $Gr = 2, Gm = 2,$ Sc=0.23, Pr=0.23, R=2, t=0.01, $\omega = 0.2$. It is noticed that the velocity decreases with the increase of dusty fluid parameter (B_1) and dust particles parameter (B_2)**.**

Temperature Profile: Figure 8 shows the effect of the parameters Pr and Q on Temperature profile at any point of the fluid in the absence of other parameters**.** It is noticed that the initially temperature falls in the increase of Prandtl number (Pr) and Source parameter (Q). then after it oscillate.

Mass Concentration Profile: Figure 9 shows the effect of the parameters Sc and R m on mass concentration profile at any point of the fluid in the absence of other parameters It is noticed that the mass concentration decreases with the increase of Schmidt number (Sc) and chemical reaction parameter (R).

Shearing Stress: Table 1 shows the effects of different parameters on Shearing stress. It is noticed that shearing stress increases in the increase of Grash off number (Gr), modified Grash off number (Gm), whereas decreases in the increase of all other parameters.

Table 1: Effect of Different Parameters on Shearing Stress

<table>
<tr><th colspan="5">When Vo=2 K=2, Q=2,$B_1 = 0.8$, $B_2 = 0.8$, and Sc=0.23</th><th rowspan="2">Shearing Stress(τ)</th><th colspan="5">When Pr=0.71, Gr=2, Gm=2, M=2, R=2 and Sc=0.23</th><th rowspan="2">Shearing Stress(τ)</th></tr>
<tr><th>Pr</th><th>Gr</th><th>Gm</th><th>M</th><th>R</th><th>Sc</th><th>B_1</th><th>B_2</th><th>Q</th><th>Vo</th></tr>
<tr><td>0.71</td><td rowspan="3">2</td><td rowspan="5">2</td><td rowspan="7">2</td><td rowspan="9"></td><td>-0.168</td><td>0.23</td><td rowspan="3">1</td><td rowspan="5">1</td><td rowspan="7">2</td><td rowspan="9">2</td><td>-0.168</td></tr>
<tr><td>2</td><td>-1.881</td><td>0.6</td><td>-1.92</td></tr>
<tr><td>3</td><td>-2.15</td><td>0.87</td><td>-2.02</td></tr>
<tr><td rowspan="8">0.71</td><td>4</td><td>-0.88</td><td rowspan="8">2</td><td>3</td><td>-6.467</td></tr>
<tr><td>6</td><td>-0.08</td><td>5</td><td>-6.473</td></tr>
<tr><td rowspan="6"></td><td>4</td><td>-0.80</td><td rowspan="6">1</td><td>3</td><td>-6.479</td></tr>
<tr><td>6</td><td>0.06</td><td>5</td><td>-6.498</td></tr>
<tr><td rowspan="4"></td><td>4</td><td>-4.21</td><td rowspan="4">1</td><td>4</td><td>-6.474</td></tr>
<tr><td>6</td><td>-6.46</td><td>6</td><td>-6.486</td></tr>
<tr><td rowspan="2"></td><td>4</td><td>-6.475</td><td rowspan="2">2</td><td>4</td><td>-7.70</td></tr>
<tr><td>6</td><td>-6.487</td><td>6</td><td>-79.11</td></tr>
</table>

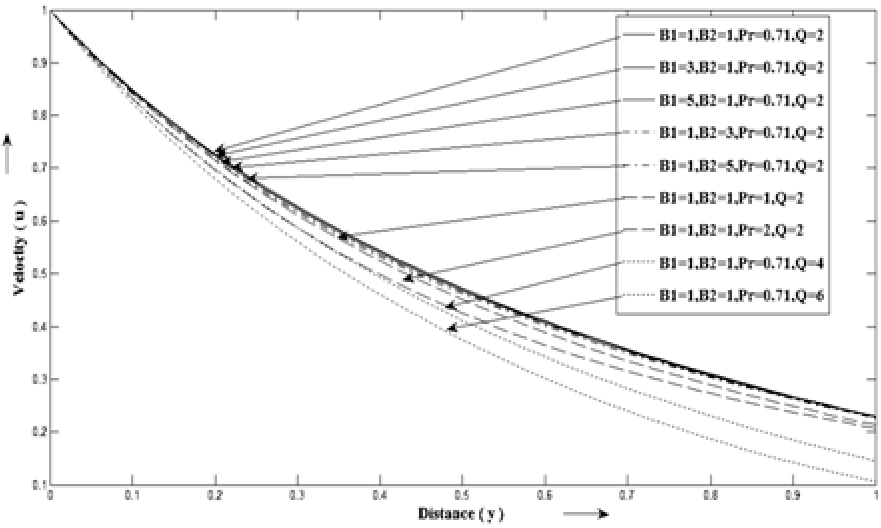

Figure 1: Effect of B_1, B_2, Pr and Q on Velocity Profile (u), when Vo=2, Sc=0.23, Gr=2, Gm=2, R=2, M=2, K=2, t=0.01, $\omega = 0.2$

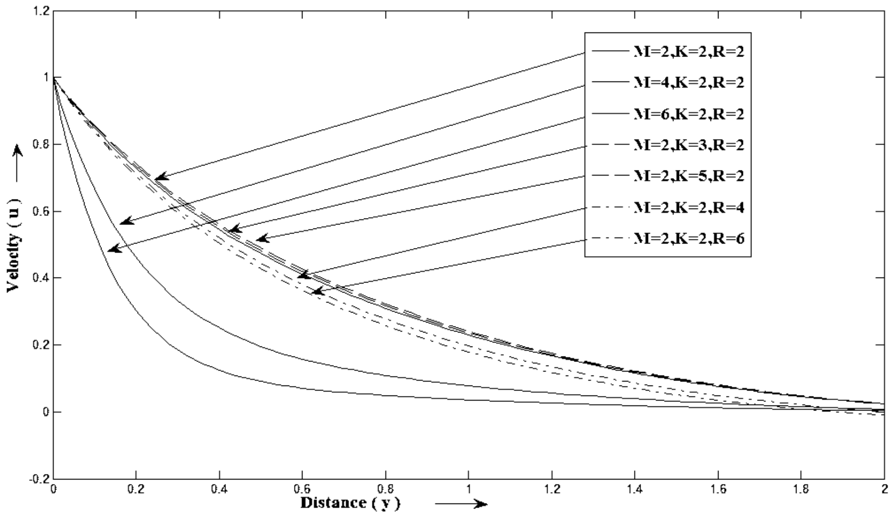

Figure 2: Effect of M, K and R on Velocity Profile (u), when $B_1 = 1$, $B_2 = 1$ $Pr = 0.23$, Q=2 Vo=2, Sc=0.23, Gr=2, Gm=2, T=0.01, $\omega = 0.2$

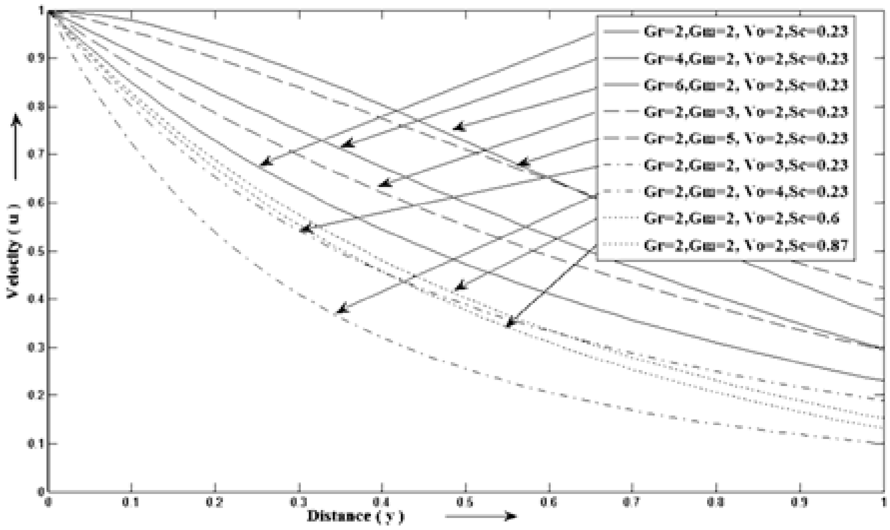

Figure 3: Effect of Gr, Gm, Vo, Sc on Velocity Profile (u), when $B_1 = 1$, $B_2 = 1$ $Pr = 0.23$, Q=2, K=0.23, M=2, R=2, T=0.01, $\omega = 0.2$

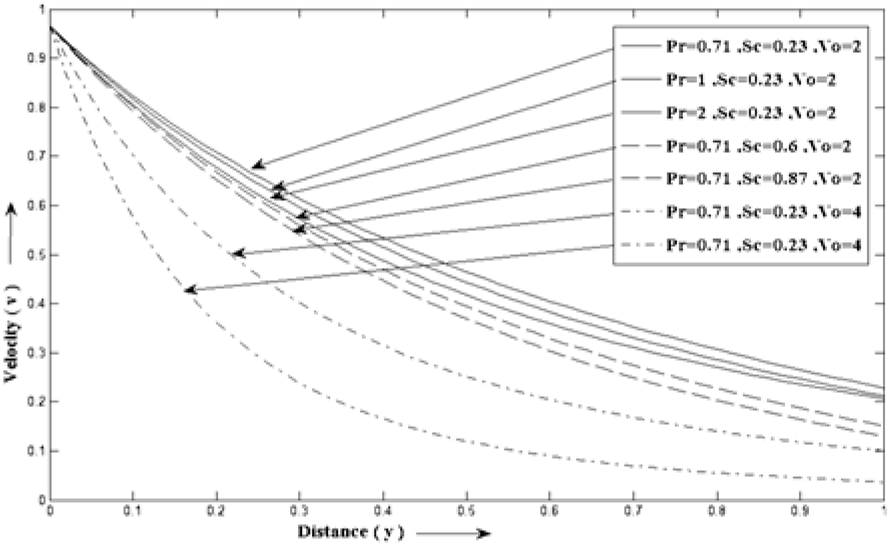

Figure 4: Effect of Pr, Vo, Sc on Velocity Profile (v), when $B_1 = 1, B_2 = 1 Gr = 2, Gm = 2$, Q=2, K=2, M=2, R=2, T=0.01, $\omega = 0.2$

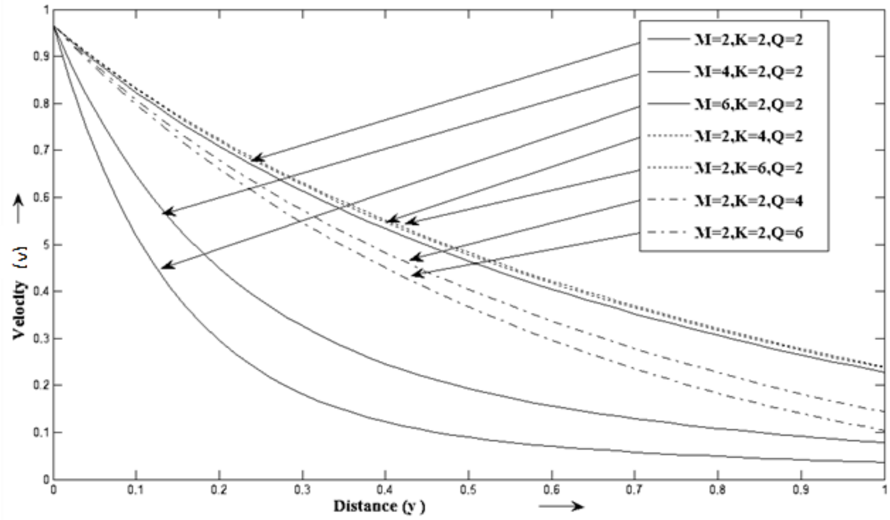

Figure 5: Effect of M, K and Q on Velocity Profile (v), when $B_1 = 1, B_2 = 1$ $Gr = 2, Gm = 2$, Sc=0.23, Pr=0.23, R=2, t=0.01, $\omega = 0.2$

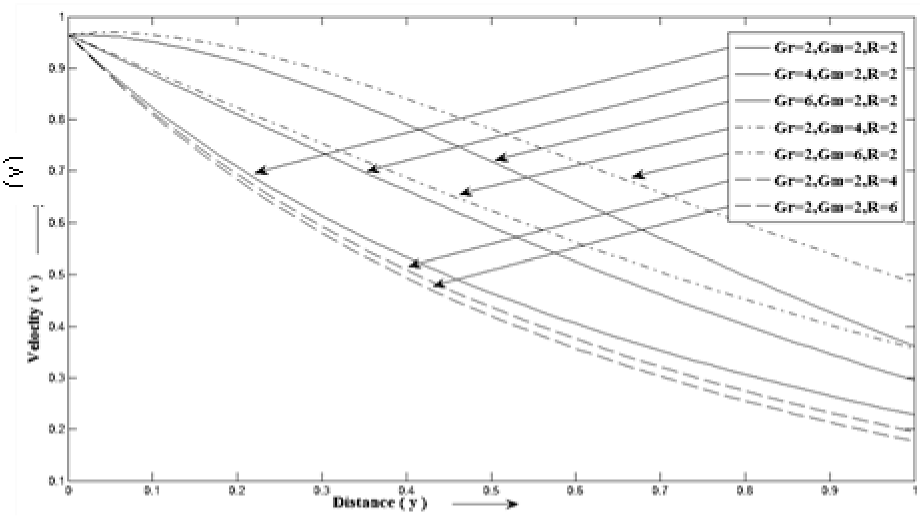

Figure 6: Effect of Gm, Gr and R on Velocity Profile (v), when $B_1 = 1, B_2 = 1$ $K = 2, M = 2$, Sc=0.23, Pr=0.23, Q=2, t=0.01, $\omega = 0.2$

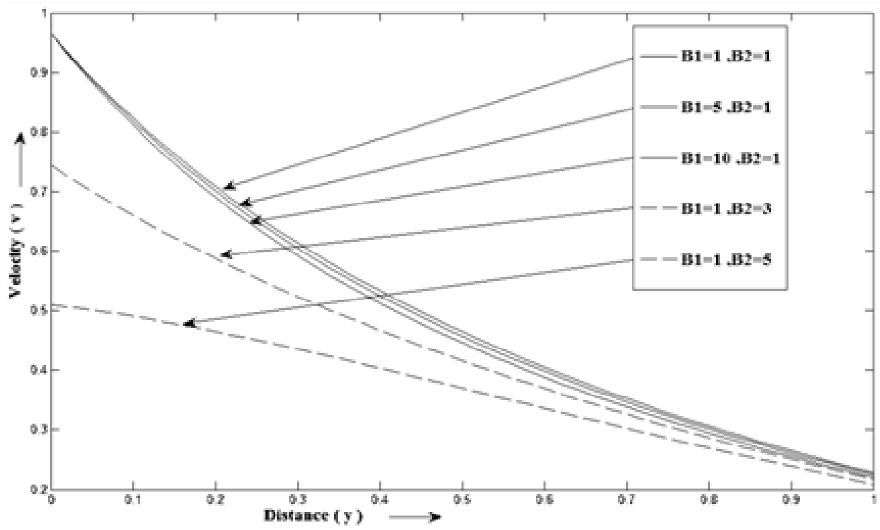

Figure 7: Effect of B_1 and B_2 on Velocity Profile (v), when M=2, K=2, R=2 $Gr = 2, Gm = 2$, Sc=0.23, Pr=0.23, R=2, t=0.01, $\omega = 0.2$

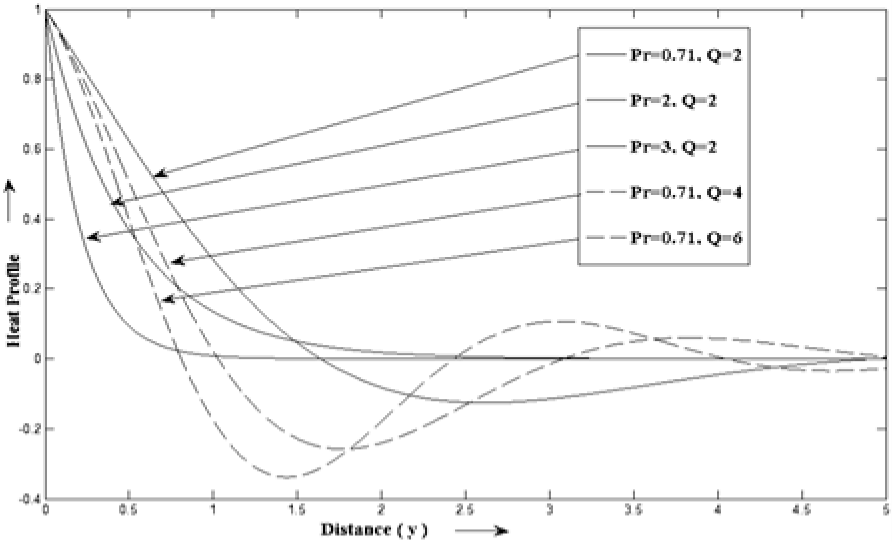

Figure 8: Effect of Pr and Q on Heat Profile (θ) in the Absence of Other Parameters

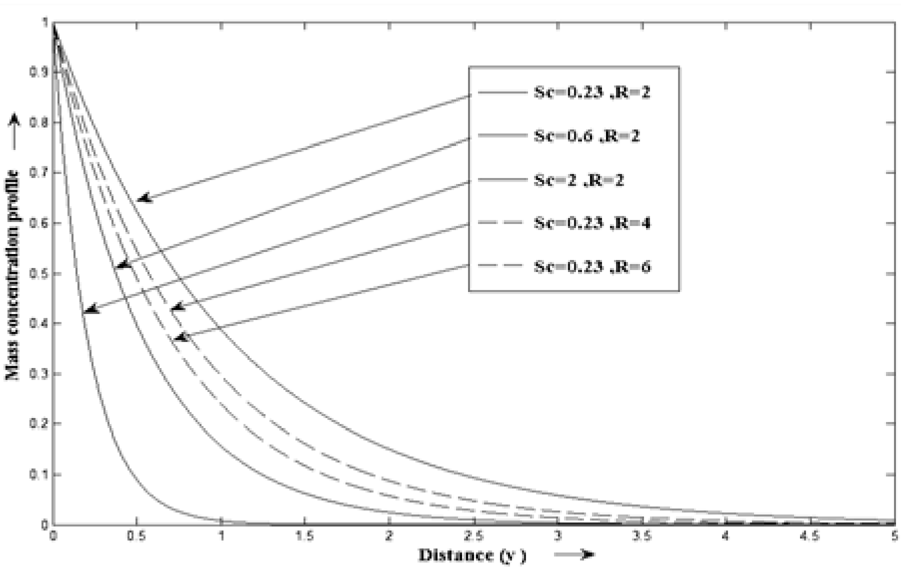

Figure 9: Effect of Sc and R on Mass Concentration Profile (C) in the Absence of Other Parameters

CONCLUSIONS

From the above dissicusion and result the following points are set up

- Both the Velocity 'u' and 'v 'at any point increases with the increase of permeability parameter porous medium Grashoff number and modified Grashoff number, where as decreases with the increase of dusty fluid parameter, dust particles parameter, Prandtl number, chemical reacting spices, magnetic parameters, source parameter Schmidt number and suction parameter.
- Mass concentration at point of the fluid decreases with the increase of Schmidt number and chemical reaction parameter.
- Grashoff number and modified Grashoff number play the key role for the increase of Shearing stress at the plate.

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