International Journal of Mechanical Engineering (IJME)
ISSN (P): 2319-2240; ISSN (E): 2319-2259
Vol. 6, Issue 3, Apr - May 2017; 9-18

International Academy of Science, Engineering and Technology
Connecting Researchers; Nurturing Innovations

NONLOCAL SHEAR DEFORMATION THEORY FOR THE VIBRATION OF A GRAPHENE SHEET RESTING ON PASTERNAK'S FOUNDATION

A.M. ZENKOUR[1] & A.H. AL-SUBHI[2]

[1]Department of Mathematics, Faculty of Science, Kafrelsheikh University, Kafrelsheikh, Egypt

[1,2]Department of Mathematics, Faculty of Science, King Abdulaziz University, Jeddah, Saudi Arabia

ABSTRACT

This article presents the vibration of a single-layered graphene sheet resting on an elastic foundation by using nonlocal first-order shear deformation theory. Equations of motion for a simply-supported graphene sheet are obtained via a nonlocal shear deformation theory. Effects of nonlocal parameter as well as length of graphene sheet, mode numbers, three-parameter of foundation and thermal parameter are discussed. A comparison example is presented to show the accuracy of the present results.

KEYWORDS: Two-Parameter Elastic Foundation, Nonlocal Elasticity, Vibration, Graphene Sheet

INTRODUCTION

Vibration analysis of nanoplates using the nonlocal theory of elasticity [1-4] is the main subject of research works in recent years. The single-layered graphene sheet (SLGS), or double-layered graphene sheet (DLGS), or multi-layered grapheme sheet (MLGS) has high resistance and unique properties. So, all of them are used for manufacturing of many devices such as oscillators, clocks and sensor devices. Sakhaee-Pour et al. [5] presented vibration analysis of SLGSs by using a molecular structural mechanics method. Pradhan and Phadikar [6] studied the nonlocal vibration of SL and DL nano-plates via classical plate theory (CPT) and first-order shear deformation plate theory (FSPT). Ansari et al. [7] investigated the vibrational behavior of a SLGS via FSPT and differential equations are solved by using generalized differential quadrature method for various boundary conditions. Pradhan and Kumar [8] presented the vibration analyses of an orthotropic SLGS using the CPT and solved the governing equations of motion by using differential quadrature method. Satish et al. [9] presented thermal vibration analyses of orthotropic nanoplates based on nonlocal continuum mechanics for small scale effects. Shen et al. [10] used Galerkin's method to present the vibration analysis of a SLGS based nano-mechanical sensor via nonlocal Kirchhoff plate theory.

Most of nanostructures are resting on two-parameter elastic foundation. Ansari et al. [11, 12] studied the vibration analysis of a MLGS using the FSPT according to Winkler-type foundation. Murmu and Adhikari [13] investigated the nonlocal vibration of bonded double nanoplate systems according to Winkler-type foundation. Wang et al. [14] used the nonlocal theory to derive the nonlinear governing equations for double-layered nanoplates subjected to four different boundary conditions according to Winkler-type foundation. Behfar and Naghdabadi [15] presented nano scale vibration analysis of a MLGS embedded in elastic medium. Chien et al. [16] investigated nonlinear vibration of laminated plates resting on a nonlinear elastic medium. Liew et al. [17] proposed a continuum-based plate model to study the vibration behavior of MLGSs that are embedded in an elastic matrix.

Pradhan and Murmu [18] employed nonlocal plate theory and used DQM for vibration analyses of nano-SLGSs embedded in elastic medium. Pradhan and Kumar [19] presented vibration analysis of orthotropic SLGS embedded in Pasternak's elastic medium.

The natural vibration frequency of a SLGS resting on a two-parameter Pasternak's foundation [20-24] is investigated in the present article. The nonlocal elasticity theory via the first-order shear deformation theory is presented. The differential governing equations are derived and their solution is analytically presented for a simply-supported SLGS. The effects of nonlocal index, two-parameter foundation on the natural vibration frequencies are illustrated. A comparison with the literature is presented and benchmark results are plotted for sensing the effect of all used parameters.

BASIC EQUATIONS

Let us consider a SLGS resting on elastic foundation and subjected to uniform load. The SLGS is of length a, width b and uniform thickness h as shown in Figure 1. The SLGS is made of a homogeneous isotropic and linearly elastic material with Young's modulus of elasticity E, Poisson's ratio ν, shear modulus G and material density ρ.

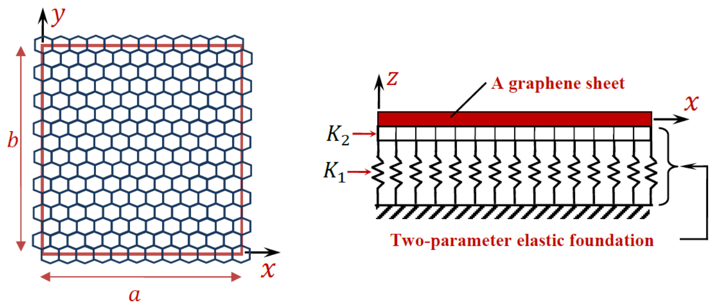

Figure 1: A Continuum Plate Model of a Single-Layered Graphene Sheet Resting on Elastic Foundation

The displacements u_j of a SLGS according to the FSPT are expressed as

$$\begin{aligned} u_1(x,y,z,t) &= u(x,y,t) + z\,\psi(x,y,t), \\ u_2(x,y,z,t) &= v(x,y,t) + z\,\phi(x,y,t), \\ u_3(x,y,z,t) &= w(x,y,t), \end{aligned} \tag{1}$$

in which u_1, u_2, and u_3 are the displacements in the x, y, and z directions; u and v are in-plane displacements while w denotes transverse displacement (deflection); ψ and ϕ are rotational displacement about y- and x-axes, respectively.

The strain-displacement equations of elasticity are given by

$$\begin{Bmatrix} \varepsilon_{xx} \\ \varepsilon_{yy} \\ \gamma_{xy} \end{Bmatrix} = \begin{Bmatrix} \frac{\partial u}{\partial x} \\ \frac{\partial v}{\partial y} \\ \frac{\partial v}{\partial x} + \frac{\partial u}{\partial y} \end{Bmatrix} + z \begin{Bmatrix} \frac{\partial \psi}{\partial x} \\ \frac{\partial \phi}{\partial y} \\ \frac{\partial \varphi}{\partial x} + \frac{\partial \psi}{\partial y} \end{Bmatrix}, \quad \varepsilon_{zz} = 0, \quad \begin{Bmatrix} \gamma_{yz} \\ \gamma_{xz} \end{Bmatrix} = \begin{Bmatrix} \phi + \frac{\partial w}{\partial y} \\ \psi + \frac{\partial w}{\partial x} \end{Bmatrix}. \tag{2}$$

The constitutive equations of an isotropic SLGS according to the nonlocal elasticity can be expressed as

$$\begin{Bmatrix} \sigma_{xx} - \xi \nabla^2 \sigma_{xx} \\ \sigma_{yy} - \xi \nabla^2 \sigma_{yy} \end{Bmatrix} = \frac{E}{1-\nu^2} \begin{bmatrix} 1 & \nu \\ \nu & 1 \end{bmatrix} \begin{Bmatrix} \varepsilon_{xx} \\ \varepsilon_{yy} \end{Bmatrix}, \quad \begin{Bmatrix} \sigma_{yz} - \xi \nabla^2 \sigma_{yz} \\ \sigma_{xz} - \xi \nabla^2 \sigma_{xz} \\ \sigma_{xy} - \xi \nabla^2 \sigma_{xy} \end{Bmatrix} = G \begin{Bmatrix} \gamma_{yz} \\ \gamma_{xz} \\ \gamma_{xy} \end{Bmatrix}, \tag{3}$$

where $\xi = (ae_0)^2$ represents the nonlocal index in which the parameter a denotes an internal characteristic length and e_0 denotes a material constant determined experimentally.

Hamilton's principle is used here to obtain the equations of motion as

$$\int_{t_1}^{t_2} \left\{ \int_{-h/2}^{h/2} \int_{\Omega} \left(\rho \frac{\partial^2 u_i}{\partial t^2} \delta u_i + \sigma_{xx}\delta\varepsilon_{xx} + \sigma_{yy}\delta\varepsilon_{yy} + \sigma_{xy}\delta\gamma_{xy} + \sigma_{xz}\delta\gamma_{xz} + \sigma_{yz}\delta\gamma_{yz} \right) \mathrm{d}\Omega\, \mathrm{d}z \right.$$

$$\left. - \int_{\Omega} \left(q - R_f\right)\delta w \, \mathrm{d}\Omega \right\} \mathrm{d}t = 0, \quad \textbf{(4)}$$

where q is the transverse distributed load and R_f is thetwo-parameter Pasternak's foundation

$$R_f = (K_1 - K_2\nabla^2)w, \quad \textbf{(5)}$$

in which ∇^2 is the Laplacian, K_1 is the linear Winkler's modulus and K_2 denotes Pasternak's (shear) foundation modulus.

Integrating the displacement gradients in ε_{ij} by parts in Eq. (4) to obtain the governing equations in the form (setting $q = 0$)

$$\begin{aligned}
\frac{\partial N_{xx}}{\partial x} + \frac{\partial N_{xy}}{\partial y} &= I_0 \frac{\partial^2 u}{\partial t^2} + I_1 \frac{\partial^2 \psi}{\partial t^2}, \\
\frac{\partial N_{xy}}{\partial x} + \frac{\partial N_{yy}}{\partial y} &= I_0 \frac{\partial^2 v}{\partial t^2} + I_1 \frac{\partial^2 \phi}{\partial t^2}, \\
\frac{\partial Q_x}{\partial x} + \frac{\partial Q_y}{\partial y} - (K_1 - K_2\nabla^2)w &= I_0 \frac{\partial^2 w}{\partial t^2}, \\
\frac{\partial M_{xx}}{\partial x} + \frac{\partial M_{xy}}{\partial y} - Q_x &= I_1 \frac{\partial^2 u}{\partial t^2} + I_2 \frac{\partial^2 \psi}{\partial t^2}, \\
\frac{\partial M_{xy}}{\partial x} + \frac{\partial M_{yy}}{\partial y} - Q_y &= I_1 \frac{\partial^2 v}{\partial t^2} + I_2 \frac{\partial^2 \phi}{\partial t^2},
\end{aligned} \quad \textbf{(6)}$$

where I_j denote mass moments of inertia defined by

$$I_j = \int_{-h/2}^{h/2} \rho z^j \mathrm{d}z. \quad \textbf{(7)}$$

and N_{ij}, M_{ij}, and Q_i denote basic components of stress resultants, stress couples and shear stress resultants. They can be obtained by integrating Eq. (3) over the thickness of the SLGS as

$$\{N_{ij}, M_{ij}, Q_i\} = \int_{-h/2}^{h/2} \{\sigma_{ij}, z\sigma_{ij}, k\sigma_{iz}\} \mathrm{d}z, \quad i,j = x,y, \quad \textbf{(8)}$$

k is the transverse shear correction factor. After using Eqs. the above stress resultants can be written in terms of the displacements as

$$\begin{Bmatrix} N_{xx} - \xi\nabla^2 N_{xx} \\ N_{yy} - \xi\nabla^2 N_{yy} \\ N_{xy} - \xi\nabla^2 N_{xy} \end{Bmatrix} = \frac{Eh}{1-\nu^2} \begin{bmatrix} 1 & \nu & 0 \\ \nu & 1 & 0 \\ 0 & 0 & \frac{1-\nu}{2} \end{bmatrix} \begin{Bmatrix} \frac{\partial u}{\partial x} \\ \frac{\partial v}{\partial y} \\ \frac{\partial v}{\partial x} + \frac{\partial u}{\partial y} \end{Bmatrix}, \quad \textbf{(9)}$$

$$\begin{Bmatrix} M_{xx} - \xi\nabla^2 M_{xx} \\ M_{yy} - \xi\nabla^2 M_{yy} \\ M_{xy} - \xi\nabla^2 M_{xy} \end{Bmatrix} = D \begin{bmatrix} 1 & \nu & 0 \\ \nu & 1 & 0 \\ 0 & 0 & \frac{1-\nu}{2} \end{bmatrix} \begin{Bmatrix} \frac{\partial \psi}{\partial x} \\ \frac{\partial \phi}{\partial y} \\ \frac{\partial \phi}{\partial x} + \frac{\partial \psi}{\partial y} \end{Bmatrix}, \quad \textbf{(10)}$$

$$\begin{Bmatrix} Q_y - \xi\nabla^2 Q_y \\ Q_x - \xi\nabla^2 Q_x \end{Bmatrix} = kGh \begin{Bmatrix} \phi + \frac{\partial w}{\partial y} \\ \psi + \frac{\partial w}{\partial x} \end{Bmatrix}, \tag{11}$$

where $D = \frac{Eh^3}{12(1-\nu^2)}$ represents bending rigidity of the SLGS.

The substitution of Eqs. (9)-(11) into Eqs. (6) yields the following nonlocal partial differential equations of motion in terms of displacements

$$D\left(\frac{\partial^2 u}{\partial x^2} + \frac{1+\nu}{2}\frac{\partial^2 v}{\partial x \partial y} + \frac{1-\nu}{2}\frac{\partial^2 u}{\partial y^2}\right) = \frac{h^2}{12}(1-\xi\nabla^2)\left(I_0\frac{\partial^2 u}{\partial t^2} + I_1\frac{\partial^2 \psi}{\partial t^2}\right), \tag{12}$$

$$D\left(\frac{1-\nu}{2}\frac{\partial^2 v}{\partial x^2} + \frac{1+\nu}{2}\frac{\partial^2 u}{\partial x \partial y} + \frac{\partial^2 v}{\partial y^2}\right) = \frac{h^2}{12}(1-\xi\nabla^2)\left(I_0\frac{\partial^2 v}{\partial t^2} + I_1\frac{\partial^2 \phi}{\partial t^2}\right), \tag{13}$$

$$kGh\left(\frac{\partial \psi}{\partial x} + \frac{\partial \phi}{\partial y} + \nabla^2 w\right) - (1-\xi\nabla^2)(K_1 - K_2\nabla^2)w = I_0(1-\xi\nabla^2)\frac{\partial^2 w}{\partial t^2}, \tag{14}$$

$$D\left(\frac{\partial^2 \psi}{\partial x^2} + \frac{1+\nu}{2}\frac{\partial^2 \phi}{\partial x \partial y} + \frac{1-\nu}{2}\frac{\partial^2 \psi}{\partial y^2}\right) - kGh\left(\psi + \frac{\partial w}{\partial x}\right) = (1-\xi\nabla^2)\left(I_1\frac{\partial^2 u}{\partial t^2} + I_2\frac{\partial^2 \psi}{\partial t^2}\right), \tag{15}$$

$$D\left(\frac{1-\nu}{2}\frac{\partial^2 \phi}{\partial x^2} + \frac{1+\nu}{2}\frac{\partial^2 \psi}{\partial x \partial y} + \frac{\partial^2 \phi}{\partial y^2}\right) - kGh\left(\phi + \frac{\partial w}{\partial y}\right) = (1-\xi\nabla^2)\left(I_1\frac{\partial^2 v}{\partial t^2} + I_2\frac{\partial^2 \phi}{\partial t^2}\right) \tag{16}$$

NONLOCAL VIBRATION FREQUENCIES

The determination of natural frequencies is of fundamental importance in design of many nano-structures. The assumed form of displacement components is expressed as

$$\begin{Bmatrix} \{u, \psi\} \\ w \\ \{v, \phi\} \end{Bmatrix} = \begin{Bmatrix} \{hu^*, \psi^*\}\cos(\lambda_m x)\sin(\mu_n y) \\ hw^*\sin(\lambda_m x)\sin(\mu_n y) \\ \{hv^*, \phi^*\}\sin(\lambda_m x)\cos(\mu_n y) \end{Bmatrix} e^{i\omega t}, \tag{17}$$

where ω represents the natural frequency for the SLGS; u^*, v^*, w^*, ψ^* and ϕ^* are arbitrary parameters;

$\lambda_m = m\pi/a$ and $\mu_n = n\pi/b$ in which m and n are the wave numbers; and $i = \sqrt{-1}$.

The substitution of the solution given above into equations of motion, for constant density, yields:

$$D\left(\lambda_m^2 + \frac{1-\nu}{2}\mu_n^2\right)u^* + \frac{1+\nu}{2}D\lambda_m\mu_n v^* = \frac{\rho h^3}{12}\omega^2[1+\xi(\lambda_m^2+\mu_n^2)]u^*, \tag{18}$$

$$\frac{1+\nu}{2}D\lambda_m\mu_n u^* + D\left(\frac{1-\nu}{2}\lambda_m^2 + \mu_n^2\right)v^* = \frac{\rho h^3}{12}\omega^2[1+\xi(\lambda_m^2+\mu_n^2)]v^*, \tag{19}$$

$$kGh(\lambda_m^2+\mu_n^2)w^* + [1+\xi(\lambda_m^2+\mu_m^2)][K_1 + (\lambda_m^2+\mu_m^2)K_2]w^*$$

$$+kG\lambda_m\psi^* + kG\mu_n\phi^* = \rho h\omega^2[1+\xi(\lambda_m^2+\mu_m^2)]w^*, \tag{20}$$

$$kGh^2\lambda_m w^* + \left[D\left(\lambda_m^2 + \frac{1-\nu}{2}\mu_n^2\right) + kGh\right]\psi^* + \frac{1+\nu}{2}D\lambda_m\mu_n\phi^* = \frac{\rho h^3}{12}\omega^2[1+\xi(\lambda_m^2+\mu_n^2)]\psi^*, \tag{21}$$

$$kGh^2\mu_n w^* + \frac{1+\nu}{2}D\lambda_m\mu_n\psi^* + \left[D\left(\frac{1-\nu}{2}\lambda_m^2 + \mu_n^2\right) + kGh\right]\phi^* = \frac{\rho h^3}{12}\omega^2[1+\xi(\lambda_m^2+\mu_n^2)]\phi^*. \tag{22}$$

The above equations of motion may be written in terms of displacement parameters w^*, ψ^*, and φ^* only as

$$([P] - \omega^2[R])\{X\} = \{0\}, \tag{23}$$

where $\{X\}=\{w^*,\psi^*,\varphi^*\}^T$ is the solution vector. The non-vanishing elements of the symmetric matrices $[P]$ and $[R]$ are expressed as:

$$P_{11}=kGh^3(\lambda_m^2+\mu_n^2)+h^2[1+\xi(\lambda_m^2+\mu_m^2)][K_1+(\lambda_m^2+\mu_m^2)K_2],$$

$$P_{12}=kGh^2\lambda_m\psi^*,$$

$$P_{22}=D\left(\lambda_m^2+\frac{1-\nu}{2}\mu_n^2\right)+kGh,$$

$$P_{13}=kGh^2\mu_n\phi^*,$$

$$P_{23}=\frac{1+\nu}{2}D\lambda_m\mu_n,$$

$$P_{33}=D\left(\frac{1-\nu}{2}\lambda_m^2+\mu_n^2\right)+kGh,$$

$$R_{11}=\rho h^3\omega^2[1+\xi(\lambda_m^2+\mu_m^2)],$$

$$R_{22}=R_{33}=\frac{\rho h^3}{12}\omega^2[1+\xi(\lambda_m^2+\mu_n^2)]. \quad \textbf{(24)}$$

The frequency equation for the SLGS is given by setting $|[P]-\omega^2[R]|=0$ to get

$$A_3\omega^6-A_2\omega^4+A_1\omega^2-A_0=0, \quad \textbf{(25)}$$

in which

$$A_0=P_{22}(P_{11}P_{33}-P_{13}^2)-P_{12}(P_{12}P_{33}-P_{13}P_{23})-P_{23}(P_{11}P_{23}-P_{12}P_{13}),$$

$$A_1=R_{11}(P_{22}P_{33}-P_{23}^2)+R_{22}(P_{11}P_{33}-P_{13}^2)+R_{22}(P_{11}P_{22}-P_{12}^2),$$

$$A_2=[P_{11}R_{22}+(P_{22}+P_{33})R_{11}]R_{22},$$

$$A_3=R_{11}R_{22}^2. \quad \textbf{(26)}$$

NUMERICAL RESULTS AND DISCUSSIONS

In this section the properties of graphene sheet are considered as [6]: modulus of elasticity $E=1.02$ TPa, Poisson's ratio $\nu=0.16$ and material density $\rho=2250\ \mathrm{kg/m^3}$. It is assumed, except otherwise stated, that $a=b=10$ nm and $h=0.34$ nm. Here, the fundamental vibration frequency ($m=n=1$) is firstly compared with the corresponding ones in the literature [6, 8, 10, 19]. For this purpose, the frequency ratio ($\widehat{\omega}=\omega^{\mathrm{NL}}/\omega^{\mathrm{L}}$) is considered where ω^{NL} represents the ftrequency calculated using nonlocal theory while ω^{L} represents the ftrequency calculated using local theory. The comparison of vibration frequency ratio of nonlocal square SLGS is reported in Table 1. An excelent agreement is appeared and the present ratio is the same as those presented in the literature.

Now, let us consider additional examples to put into evidence the effect of length a, the foundation parameters K_1 and K_2 and the nonlocal index on the vibration of the present SLGS. It is to be noted that, we can get the local thermal vibration of the SLGSs by setting $\xi=0$ in the preceeding equations.

Table 1: Comparison of Fundamental Vibration Frequency Ratio $\widehat{\omega}$ of Nonlocal Square SLGS ($a = b = 10$ nm and $h = 0.34$ nm)

ξ (nm^2)	Present	[6]	[8]	[10]	[19]
0	1	1	1	1	1
1	0.91386	0.9139	0.9139	0.9139	0.9139
2	0.84673	0.8467	0.8466	0.8467	0.8468
3	0.79251	0.7925	0.7926	0.7925	0.7926

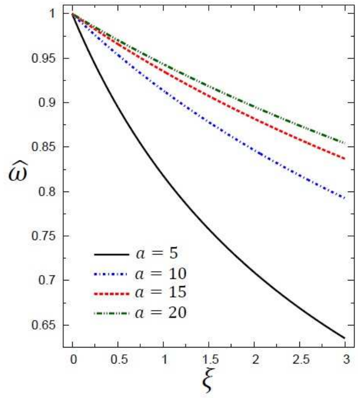

Figure 2: Fundamental Frequency Ratio of a SLGS Vs Nonlocal Index and its Length ($b = 10$nm, $h = 0.34$nm, $K_1 = 0$, $K_2 = 0$)

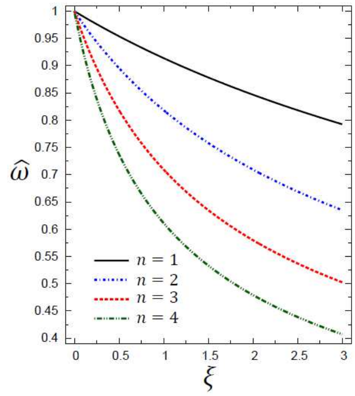

Figure 3: Natural Frequency Ratio of a Square SLGS Vs Nonlocal Index ($a = b = 10$Nm, $h = 0.34$Nm, $m = 1$, $K_1 = 0$, $K_2 = 0$)

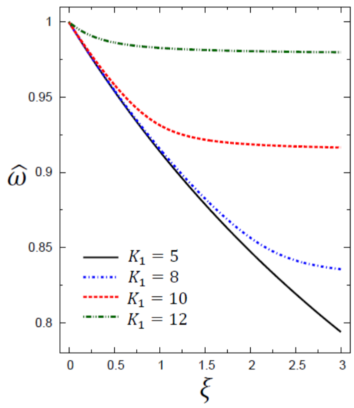

Figure 4: Fundamental Frequency Ratio of a Square SLGS Vs Nonlocal Index and Winkler's Parameter ($h = 0.34$nm, $a = b = 10$nm, $K_2 = 5$)

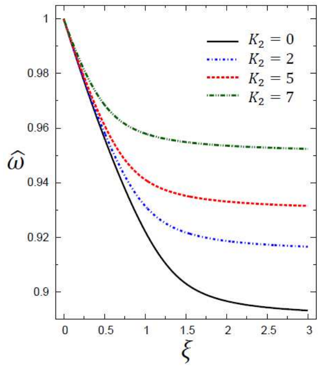

Figure 5: Fundamental Frequency Ratio of a Square SLGS Vs Nonlocal Index and Pasternak's Parameter ($h = 0.34$ nm, $a = b = 10$ nm, $K_1 = 10$)

Figure 2 shows the fundamental frequency ratio $\widehat{\omega}$ vs nonlocal index ξ for different values of the length a of the SLGS without elastic foundation ($K_1 = K_2 = 0$). The frequency ratio $\widehat{\omega}$ increases as a increases and as ξ decreases. The maximum fundamental frequency ratio occurs when $\xi = 0$. Figure 3 shows the natural ($m = 1$ and $n = 1,2,3,4$) frequency ratios $\widehat{\omega}$ vs nonlocal index ξ of the SLGS without elastic foundation ($K_1 = K_2 = 0$). The natural frequency ratio $\widehat{\omega}$ increases as n decreases.

Figures 4 and 5 show the fundamental frequency ratio $\widehat{\omega}$ of a square SLGS vs nonlocal index ξ for different values of Winkler's parameter K_1 and Pasternak's parameter K_2, respectively. Once again, the fundamental frequency ratio $\widehat{\omega}$ increases as ξ decreases. The maximum frequency ratio occurs when $\xi = 0$ and for all values of K_1 and K_2. The frequency ratio is increasing with the increase of Winkler's parameter K_1 and Pasternak's parameter K_2.

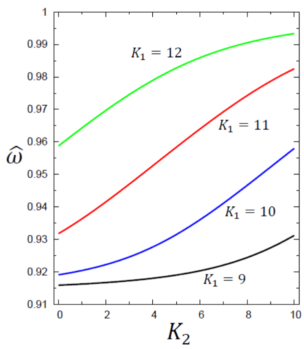

Figure 6: Fundamental Frequency Ratio of a Square SLGS Vs Pasternak's Parameter ($h = 0.34$ nm, $a = b = 10$ nm, ξ =1)

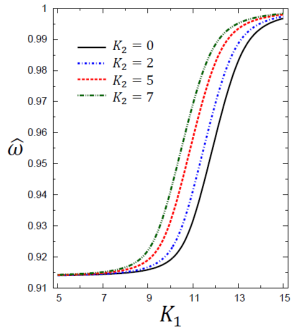

Figure 7: Fundamental Frequency Ratio of a Square SLGS Vs Winkler's Parameter ($h = 0.34$ Nm, $a = b = 10$ Nm, ξ =1)

The plots of fundamental frequency ratio $\hat{\omega}$ of a square SLGS vs the elastic foundation parameters K_1 and K_2 are appeared in Figures 6 and 7. The frequency ratio is independent of K_2 for small values of K_1. For constant values of K_1 the frequency ratio is increasing as K_2 increasing (Figure 6). Also, for constant values of K_2 the frequency ratio is rapidly increasing as K_1 increasing (Figure 7), especially in the range $8 < K_1 < 14$.

CONCLUSIONS

The nonlocal vibration frequencies of a single-layered graphene sheet resting on two-parameter Pasternak's foundation are investigated. The local-to-nonlocal vibration frequency ratios are compared well with the corresponding results in the literature. The fundamental and natural frequency ratios are presented to serve as benchmarks for future comparisons. It can be observed some novel phenomena from the discussion of the results. The results are very sensitive to the variation of nonlocal index. The inclusion of two-parameter Pasternak's foundation is very pronounced. The maximum frequency ratio occurs when neglecting the nonlocal index.

REFERENCES

1. A.C. Eringen, Nonlocal polar elastic continua, *Int. J. Eng. Sci.* 10, 1–16, 1972.
2. A.C. Eringen and D.G.B. Edelen, On nonlocal elasticity, *Int. J. Eng. Sci.* 10, 233–248, 1972.
3. A.C. Eringen, On differential equations of nonlocal elasticity and solutions of screw dislocation and surface waves, *J. Appl. Phys.* 54, 4703–4710, 1983.
4. A.C. Eringen, *Nonlocal Continuum Field Theories*, New York: Springer Verlag, 2002.
5. A Sakhaee-Pour, M T Ahmadian1 and R Naghdabadi, Vibrational analysis of single-layered graphene sheets, *Nanotechnology* 19(8), 085702, 2008.
6. S.C. Pradhan and J.K. Phadikar, Nonlocal elasticity theory for vibration of nanoplates, *J. Sound Vib.* 325, 206–223, 2009.
7. R. Ansari, S. Sahmani and B. Arash, Nonlocal plate model for free vibrations of single-layered graphene sheets, *Phys. Lett. A* 375, 53–62, 2010.
8. S.C. Pradhan and A. Kumar, Vibration analysis of orthotropic graphene sheets using nonlocal elasticity theory and differential quadrature method, *Compos. Struct.* 93, 774–779, 2011.
9. N. Satish, S. Narendar and S. Gopalakrishnan, Thermal vibration analysis of orthotropic nanoplates based on nonlocal continuum mechanics, *Phys. E* 44, 1950–1962, 2012.
10. Z-B. Shen, H-L. Tang, D-K. Li and G-J. Tang, Vibration of single-layered graphene sheet-based nanomechanical sensor via nonlocal Kirchhoff plate theory, *Comput. Mater. Sci.* 61, 200–205, 2012.
11. R. Ansari, R. Rajabiehfard and B. Arash, Nonlocal finite element model for vibrations of embedded multi-layered graphene sheets, *Comput. Mater. Sci.* 49, 831–838, 2010.
12. R. Ansari, B. Arash and H. Rouhi, Vibration characteristics of embedded multi-layered graphene sheets with different boundary conditions via nonlocal elasticity, *Compos. Struct.* 93, 2419–2429, 2011.

13. T. Murmu and S. Adhikari, Nonlocal vibration of bonded double-nanoplate-systems, *Compos. B* 42, 1901–1911, 2011.

14. Y. Wang, F. Li, X. Jing and Y. Wang, Nonlinear vibration analysis of double-layered nanoplates with different boundary conditions, *Phys. Letters A* 379, 1532–1537, 2015.

15. K. Behfar and R. Naghdabadi, Nanoscale vibration analysis of a multi-layered graphene sheet embedded in an elastic medium, *Compos. Sci. Tech.* 65, 1159–1164, 2005.

16. R-D. Chien and C-S. Chen, Nonlinear vibration of laminated plates on a nonlinear elastic foundation, *Compos. Struct.* 70, 90–99, 2005.

17. K.M. Liew, X.Q. He and S. Kitipornchai, Predicting nano vibration of multi-layered graphene sheets embedded in an elastic matrix, *Acta Mater.* 54, 4229–4236, 2006.

18. T.Murmuand and S.C. Pradhan, Vibration analysis of nano-single-layered graphene sheets embedded in elastic medium based on nonlocal elasticity theory, *J. Appl. Phys.* 105, 064319, 2009.

19. S.C. Pradhan and A. Kumar, Vibration analysis of orthotropic graphene sheets embedded in Pasternak elastic medium using nonlocal elasticity theory and differential quadrature method, *Comput. Mater. Sci.* 50, 239–245, 2010.

20. A.M. Zenkour, The refined sinusoidal theory for FGM plates on elastic foundations, *Int. J. Mech. Sci.* 51 (11-12), 869–880, 2009.

21. A.M. Zenkour, Hygro-thermo-mechanical effects on FGM plates resting on elastic foundations, *Compos. Struct.* 93 (1), 234–238, 2010.

22. A.M. Zenkour, Bending of orthotropic plates resting on Pasternak's foundations by mixed shear deformation theory, *Acta Mech. Sin.* 27(6), 956–962, 2011.

23. A.M. Zenkour, Thermal bending of layered composite plates resting on elastic foundations using four-unknown shear and normal deformations theory, *Compos. Struct.* 122, 260–270, 2015.

24. A.M. Zenkour, Free vibration of a microbeam resting on Pasternak's foundation via the GN thermoelasticity theory without energy dissipation, *J. Low Freq. Noise, Vib. Active Contr.* 35 (4), 303–311, 2016.