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QUASI-SCMODULES

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ABSTRACT

The concept of quasi-semi prime cancellation (for short quasi-SC) module which a generalization the concept of semi prime cancellation module. Also, many properties and several results about this concept have been proved.

KEYWORDS: *Semi Prime Ideal, Cancellation Module, Trace of Module, Multiplication Module, Flat Module*

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INTRODUCTION

Let R be a commutative ring and M be the R-module. Gilmer [1, p. 60] has been defined the concept of a cancellation ideal to be the ideal I of R which satisfies the following

Whenever AI = BI with A and B are ideals of R implies A=B

Mijbass in [2] has been generalized this concept of modules. He has been defined the cancellation of modules as follows:

The R-module M is called a cancellation module whenever AM = BM with A and B are ideals of R implies A = B.

In this work we shall introduce the concept of the Semi prime cancellation module by using some restrictions on the ideals A and B in the above definition, namely we shall say that.

The R-module M is called Semi prime cancellation, whenever AM = BM with A is a Semi prime ideal of R and B is ideal of R implies A = B.

Mijbass and Bothyna N. Shihab in [3] has been put condition on the concept of cancellation module which was named by restricting cancellation module and defined as follows: The R-module M is called a restricted cancellation module whenever AM=BM and AM≠0with A and B are ideals of R, then A=B. [3].

An ideal A of R is said to be semi prime if A=$\sqrt{A}$. This paper consists of two sections.$\mathcal{S}_1$: study the semiprime cancellation module in the class of multiplication module. Many important results are provided. In $\mathcal{S}_2$the concept of quasi-semiprime cancellation (for short quasi-SC) module hasd been introduced and defined as follows: -Let M be the R-module. Then M is called weak purely cancellation module if AM = BM, where A is a pure ideal of R and B is any ideal of R, then A + ann(M) = B + ann(M).

This concept is a generalization of the semi prime cancellation module and this section contains many properties and several results.

STUDY OF THE SEMIPRIME CANCELLATION PROPERTY IN THE CLASS OF THE MULTIPLICATION

Recall that the R-module M is said to be a multiplication module if for every sub module N of M, there exists an ideal Iof R such that N =IM [4].Several results of this relation have been represented.

Now, we give the following proposition.

Proposition (2.1)

If M is a multiplication R-module. N is Semi prime cancellation sub module, then M is a Semi prime cancellation module.

Proof: We have N is a sub module of M and M is multiplication R-module, that is N = JM, where J is an ideal of R. Let AM = BM, where A is Semi prime ideal of R and B is an ideal of R. Then AJM = BJM which implies AN = BN and hence A = B (since N is a Semi prime cancellation module). The proof is complete.

Next, we have the following results:

Proposition (2.2)

Let M be a multiplication Semi prime cancellation R-module, N is a sub module of M. Then the following are equivalents:

- N is a Semi prime cancellation module.
- N: M) is Semi prime cancellation ideal of R. (2)
- N = AM, where A is Semi prime ideal of R and satisfies the property of Semi prime cancellation.

Proof: (1) ((2) Suppose that N is Semi prime cancellation and let A (N: M) = B (N: M), where A is Semi prime ideal of R and B is an ideal of R. Then A(N: M) M = B (N: M) M which implies AN = BN. Hence A = B. Therefore (N: M) is Semi prime cancellation ideal of R.

2) ⇒ (3) put A = (N:M).(

(3)⇒(1) Let CN = DN, where C is Semi prime ideal of R and D is any ideal of R. Then CAM = DAM by (3). Thus CA = DA (since M is cancellation module). Therefore C = D by (3). Hence N is Semi prime cancellation module.

A sub module N of an R-module M is said to be pure if IM

∩N = IN, for every ideal I of R.

In case R is PID or M is cyclic, then N is pure if and only if rM∩ N = rN, ∀ r ∈R, [5]

We end this section by the following result.

Proposition (2.3)

Let M be the R-module, N is asemiprimesubmodule of M satisfies the property of semi prime cancellation. Then M is a Semi prime cancellation module.

Proof: Let AM = BM, where A is a pure ideal of R and B is an ideal of R. We have N as a pure sub module, then $N \cap AM=AM$

and $N \cap BM = BM$. Thus AN = BN and hence A = B (since N is a semi prime cancellation module). Therefore M is a semi prime cancellation module.

QUASI-SC MODULES

As a generalization of semi prime cancellation property in the modules we shall introduce the concept of weak semi prime cancellation modules. In this section we shall discuss the results that we have obtained in section one.

We start with the following definition.

Definition (3.1)

Let M be the R-module. Then M is called a Quasi-SC module if AM = BM, where A is a semi prime ideal ofR and B is an ideal of R, then A + ann(M) = B + ann(M).

Remark (3.2):-

Every semi prime cancellation module is a Quasi-SC module.

The converse of remark (2.2) is not true, as it is seen by the following example:

Example (3.3)

Consider Z2 as a Z-module and let m1=$\bar{1} \in$ Z2 and m2 =$\bar{3} \in$ Z4,

since m1 = m2 and ann(Z2) = (2). Now, (1) + ann(Z2) = (3) + ann(Z2) = Z4. Therefore Z2 is a Quasi-SCZ4-module. But Z2 is not a semi prime cancellation module, see examples and remark ((1.2), 5).

The converse of remark (2.2) holds under the condition M is faithful.

Proposition (3.4)

If M is a faithful Quasi-SC module, then M is a semi prime cancellation module.

Proof: It is trivial, so it is omitted.

In the following proposition we shall prove that the class of cyclic modules is contained in the class Quasi-SC modules.

Proposition (3.5)

Every cyclic module is a Quasi-SC module.

Proof: Let M = <m> be a cyclic module over R with $m \in M$

and let A<m> = B <m>, where A is a semi prime ideal in R and B is an ideal in R. Then am $\in$ B<m>, a $\in$ A, implies a m = bm, where b $\in$ B. Therefore a m – bm = 0, implies (a – b)m=0

Then a – b $\in$ ann(M). But a = b + a – b. Therefore a $\in$ B + ann (M), implies A $\subseteq$ B + ann(M).

Then A + ann (M) $\subseteq$B + ann (M).

Similarly we can prove that B + ann (M) $\subseteq$ A + ann (M) and hence A + ann (M) = B + ann (M), which is the required value.

We shall give same characterizations of a Quasi-SC modules in the following proposition.

Theorem (3.6)

Let M be an R-module. Then the following statements are equivalent:

(1) M is a Quasi-SC module.

(2) If AM $\subseteq$ BM, such that A is an ideal of R and B is a pure ideal of R then A $\subseteq$ B + ann (M).

(3) If <a>M $\subseteq$BM, such that a $\in$ R and B is a semi prime ideal of R, then a $\in$ B + ann (M).

(4) (AM:M) = A + ann (M), for all semi prime ideals A of R.

(5) (AM:BM) = (A + ann (M):B), where A is a semi prime ideal of R. and B is an ideal of R.

Proof: It is easy and clear.

Proposition (3.7)

Let M and N be two R-modules and L =$\sum_{\alpha\epsilon\gamma}\theta_\alpha$(M) be

A sub modules of N where the sum is taken for any subsetγ

of Hom (M,N), L is Quasi-SC and ann (L) = ann(M).

Then M is a Quasi-SC module.

Proof: Let AM = BM, where A is semi prime ideal of R and B is any ideal of R. Then θ_α(AM) = θ_α(BM), implies

$\sum_{\alpha\epsilon\gamma}\theta_\alpha$(AM) =$\sum_{\alpha\epsilon\gamma}\theta_\alpha$ (BM). But θ_α(AM) = Aθ_α(M) =θ_α(BM)=Bθ_α(M).

Then A$\sum_{\alpha\in\gamma}\theta_\alpha$(M) = B$\sum_{\alpha\in\gamma}\theta_\alpha$(M).

Therefore AL = BL (since L is Quasi-SC module), implies A +ann(L) = B + ann(L). Therefore Aann(M) = B + ann(M). Then M is a Quasi-SC module.

Corollary (3.8)

If M is the R-module, T(M) is a Quasi-SC ideal in R and ann(T(M)) = ann(M). Then M is a Quasi-SC module.

Proof: The result is clear by using proposition (3.7) and the definition of T(M).

The dual of a module will be Quasi-SC whenever the trace of the module satisfies this property, as it is shown in the following result.

Proposition (3.9)

If M is the R-module and T(M) is a Quasi-SC module, and ann(T(M)) = ann(M). Then M is a Quasi-SC module.

Proof: It is obvious.

Proposition (3.10)

If M is a multiplication of R-module, N is a sub module of M such that ann R (N) = ann R (M) and N is a Quasi-SC, then M is Quasi-SC module.

Proof: Let AM = BM, where A is a pure ideal of R and B is an ideal of R. Then AIM = BIM (since M is multiplication).

CONCLUSIONS

- The class of Quasi-Sc modules is a generalization of the concept of semi prime modules where the first is considered the weakest concept of the second (semi prime module). Also, we give an example to show that .
- Every Quasi-Sc modules is cancellation module but the converes is not true in general.
- Some characterizations and results about the concept of Quasi-Sc modules have been proved.

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