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CRACKING AND TORSIONAL DUCTILITY BEHAVIOR OF HSC BEAMS

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ABSTRACT

Experimental investigation was carried out to study the Torsional behavior of the Medium strength concrete (MSC) and High strength concrete (HSC) beams with the mix proportion of the concrete M50 –M80 grade and more than M100 grade. Nine NSC and Nine HSC beams with constant width (100 mm), and depth (100 mm) with effective span 800mm with varying longitudinal and transverse reinforcement ratio were tested under the standard testing conditions. A special arrangement was fabricated to apply the torque to the beams. The beams were tested under standard torsional loading procedure. The parameters studied in this investigation are ductility behavior, cracking torsional strength, ultimate torsional strength, failure pattern, Torque-rotation behavior, torsional stiffness and strains. The results obtained from the experiment were compared with the different codal equations and also the equations given by researchers. Based on these observations, conclusions were drawn. A parametric analysis was also presented for the 95 data collected from previous investigations on this topic.

KEYWORDS: HSC, NSC, Crack, Failure, Ductility, Torsion, Strength, Deformation

INTRODUCTION

Development of concrete technology resulted in several concretes with different engineering properties which attracted engineers and researchers to explore more. Fibre Reinforced Concrete (FRC), High Performance Concrete (HPC), High Strength Concrete (HSC), Self Compaction Concrete (SCC), High Density Concrete etc are the result of several research programs. HSC is often considered a relatively new material over Normal Strength Concrete (NSC), its development has been gradual over many years. As the development has continued, the definition of HSC has changed. In the 1950s, concrete with a compressive strength of 34 MPa was considered HSC.

The ACI committee has defined HSC as concrete of normal weight aggregates having compressive strength for design of 41 MPa or greater. It is possible to obtain HSC using the locally available materials by adding chemical and mineral admixtures. The chemical admixtures are added to increase the workability and the mineral admixtures are added to enhance the microstructure of the concrete. It has been observed from the literature that, HSC up to 200 MPa are produced using higher dosage of acrylic based polymers as chemical admixture and using silica fume as the mineral admixture.

The advantages of using HSC are known to have more durable properties and better corrosion resisting properties than the NSC. Higher compressive strength of concrete results in a higher modulus of elasticity and thus improves serviceability. HSC provides a better solution to reduce sizes and weight of the concrete structural elements. The studies under fracture mechanics principles revealed that there are some differences in cracking and failure behaviour of NSC and

HSC. Change in the cracking and failure behaviour changes the mechanical properties of HSC. The behaviour of the structural members mostly depends on the mechanical properties of the concrete.

Bending, shear and torsion are the most common types of failure encountered in reinforced concrete beams. Although we can calculate the safety of beams with respect to bending failures with a fair degree of certainty, the same cannot be said in regard to torsional failures. The mechanism of torsional failure is as yet not clearly understood and all formulas developed for the calculation of the torsional strength of the reinforced concrete beams are either wholly or partly empirical. This is due to the lack of rationality in our approach to the problem of torsion. Most current codes of practice uses the theoretical models for determination of the torsional strength.

These theoretical models are space truss model, skew bending theory, thin wall theory and lattice model. In the development of concrete technology the compressive strength of the concrete has reached 100MPa and higher, in the field of ready mix concrete. Since the mechanical properties of concrete are changed in high strength concrete (HSC), re-evaluation of the prediction model are necessary to reliably estimate the torsional strength of the beams made with normal and HSC. Moreover because of the wider range of concrete used, more accurate prediction of torsional strength of reinforced concrete members are required. Further, HSC is being used for vital infrastructure such as nuclear power plant structures, bridges, tall buildings.

LITERATURE REVIEW

Hossain et. al. [1] studied theoretical models used for the determination of Torsional strength of NSC and HSC. The theoretical models such as skew bending theory, space truss analogy and softened truss theory model. In the assessment with Australian code AS3600. The basic approach to skew bending theory is that the failure of rectangular section in torsion occurs by bending about an axis which is parallel to the wider face of the section and inclined at about 45 degrees to the longitudinal axis of the beam. In the space truss model the torsion is resisted by compression diagonals which consists of the concrete between cracks that spiral around the beam at constant angle. Softened truss model Similar to space truss model expect that it utilizes the full concrete cross-section and takes the softening of the concrete in to consideration.

The softened concrete is assumed to be effective and its effective transverse component which is used to predict the Torsional behavior of the concrete. It was concluded that. Higher cracking load and higher Torsional capability for a given cross-section are obtained using HSC both from experimental and theoretical models. Among the 3 theoretical models used softened truss model give the best estimate of the ultimate torsional strength of the test beams. The AS3600-2001 gives an unconservative prediction for the ultimate Torsional strength of both normal and HSC beams. HSC provided higher torsional strength than NSC with same reinforcement.

I-Kuang Fang and Jyh-Kun Shiau[2] carried out the experiment on NSC (f'_c =35MPa) and HSC (f'_c =70MPa) each 8 beams with cross-section of 350x100x3100 mm and various amount of Torsional reinforcement were tested for pure torsion. Parameters studied are cracking strength, ultimate strength and ductility. By spalled truss model and softened truss model, He concludes that, HSC provided higher Torsional strength than NSC. For beams designed with the same amount reinforcement, the ACI- 318.02 code underestimates the Torsional strength of HSC beams. The assumptions of the orientation of compression diagonals in the provision of ultimate Torsional strength of the concrete ACI- 318.02 code is not altered by amount of reinforcement in the longitudinal and Torsional effect. In general the HSC beams had similar ductile behavior to the NSC beams. The past peak strength relatively steeper for HSC with heavier reinforcement.HSC beams had higher torsional stiffness before and after cracking than NSC.

Rasmussen L. J. and Baker G. [3,13] Reported on the behavior of reinforced NSC and HSC beams subjected to pure torsion. The test series consisted of 12 totally over-reinforced beams, with parameters that influence torsional capacity and concrete strength as the only variable .Therefore, the cross-sectional dimensions and the strength and dimensions of reinforcement, were constant for all beams. The concrete strength varied between 36 and 110MPa. The test series has shown the advantage in using HSC. In addition to a higher cracking load and higher ultimate torsional capacity, use of HSC for a given cross sectional and given torque results in higher torsional stiffness, lower crack width, and lower reinforcement stresses compared to NSC. Pure torsion only occurs infrequently in practice. Normally it arises as a combined action, often with bending. However, in bridges torsion constitutes a significant design action because of eccentric forces. Since large bridge construction is obvious application for HSC, an investigation of reinforced HSC beams subjected to pure torsion is of interest.

Abdel wahid Hago, et. al. [4] proposed a direct design of reinforced concrete beams under combined bending, shear and torsion. The stress distribution in the beam was obtained using a 3-D finite element analysis of the "Design Direct Method" (DDM) to beams subjected to combined bending, shear and torsion. The approach was verified by comparison with beams designed by BS8110 (1985) and ACI 318-83 codes, in terms of economy in use of steel, and conclusions were drawn in favor of the proposed design procedure. The effect of the simultaneous application of bending, shear and torsion easily is examined by means of interaction surfaces. Designs based on the proposed DDM yield more saving in steel than that provided by the present codes provided by present codes of practice BS8110 and ACI318. Further investigation into the ultimate and service behavior of beams designed by this method is currently underway.

Hao-Jan Chiu, et. al. [5] conducted an experimental investigation on the behavior of thirteen HSC and NSC full size beams with relatively low amounts of torsional reinforcement. The crack patterns, the maximum crack widths at service load level, torsional strength, torsional ductility, and post-cracking reserve strength results of the experiments are discussed. The main parameters include the volumetric ratio of torsional reinforcement, the compressive strength of the concrete, and the aspect ratio of the cross section. It was found that the adequate of the post cracking reserve strength for specimens with relatively low amounts of torsional reinforcement is primarily related to the ratio of the transverse to the longitudinal reinforcement factor in addition to the total amount of torsional reinforcement[6,9,10].

The following conclusions are drawn. The torsional cracking strength of the specimens with hollow sections was smaller than those of the specimens with solid sections. The increase of the aspect ratio of the cross section decreases the cracking and ultimate strength and increases the crack widths for the specimens with approximately the same amount of reinforcement. The selection of equal percentage in the transverse and longitudinal directions provides adequate post cracking reserve strength, the torsional ductility needed and also the crack width control necessary at service load.

RESEARCH SIGNIFICANCE

To carry out Justification of Torsional moment strength prediction by researchers and codal provisions for NSC & HSC beams with reinforcement. The analysis and understand the Torsional behavior of NSC and HSC beams without using mineral admixtures were to understand the Torsional behavior of the beams with the variation of longitudinal &

transverse reinforcement, the cracking torsional moment w.r.t to the ultimate Torsional moment. No investigations are available on the effect of the ductility of the reinforcement. It has been observed that there is a difference in the failure pattern between NSC and HSC beams. Analysis was also carried out to understand this behavior using the data from available literature.

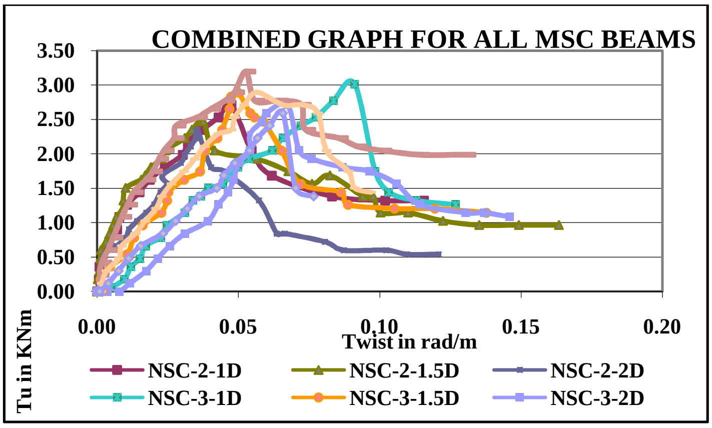
EXPERIMENTAL INVESTIGATION

In the present investigation, 9 MSC & 9 HSC beams with variation of transverse and longitudinal reinforcement were cast and tested with MSC of M80 mix & HSC of M100 mix. The 9 beams MSC and HSC each with 100 mm breadth and 100 mm depth were made into 3 series, each series had 3 beams. In each series l/d ratio kept constant with varying longitudinal reinforcement ratio as 2%, 3%, 4% & transverse reinforcement spacing as 1.0D, 1.5D, 3D. The first letter indicates the name of the beam, the second number indicates the longitudinal reinforcement ratio, and the third indicates the spacing of the transverse reinforcement with respect to the depth of the beam. To effective length of the specimen was 800mm. The total span provided was 1200mm, 200mm bearing on each side.

To avoid the failure of the specimen at the support section the support was provided with higher amount of reinforcement than the test region. To attempt the same transverse reinforcement is provided with half the spacing than the test region. Also the longitudinal reinforcement was increased comparatively. The maximum Torsional reinforcement spacing in most of the codes is limited to 0.5d or 0.75d or 300 mm whichever is less. It is not possible to understand the Torsional behavior of beams with minimum Torsional reinforcement with the above condition therefore the above codal provisions are ignored in the present investigation.

Torsional Moment v/s Twist Curves combined for all MSC and all HSC beams

From the graphs it was observed that, the torsional moment v/s twist curves are linear up to cracking and after that the non-linear behavior of curves takes place. The curves for all MSC and HSC beams are shown in Figure 1 and Figure2 respectively.



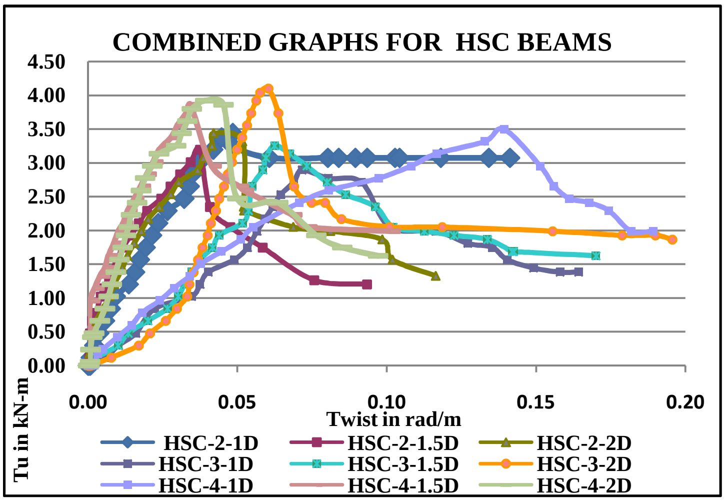


Figure 1 & 2: Torsional Moment V/S Twist Curve for all MSC & HSC Beams

All the curves behave in the similar manner till the failure of concrete. For the beams with less transverse reinforcement spacing the curves are more steeper compared to the other beams having more transverse reinforcement spacing. As the amount of longitudinal reinforcement increases the torsional moment carrying capacity also increases. From the curves obtained for MSC beams, there was sudden fall after linear variation, it denotes that the failure of concrete. It was continued to fall to some extent till the concrete fails completely. Once the contact between the concrete cracks widens the twist rapidly increases and it was continued to increase with the equal amount of torsional moment, indicating the ductility of the beam. All the curves make an angle of 45^0 approximately to the horizontal axis. The same was applicable to the HSC beams also. The failure was brittle compared to the MSC beams. The slope of the curves was more steeper compared to the MSC beams.

ANALYSIS OF THE TEST DATA BY DIFFERENT CODES

The ultimate Torsional strength of reinforced concrete beams were calculated using different theories and codes. For the 95 data collected from 7 researchers experiment were analyzed. The ultimate Torsional strength of the reinforced concrete beam is mainly depends on the strength of the concrete. The cracking torsional moment is the main parameter to know the safe Torsional moment. As the cracked section is not permitted in many codes, and also aesthetically it doesn’t look good. In the flexure and shear behavior the reserve strength is available after the first crack. The compression zone has the ability to carry the redistributed loads and moments after the propagation of cracks in tension zone.

The beam failure takes place when the compression zone also fails to take further load due to the cracks or steel yields. But it’s not true for the Torsional behavior, as the cracks propagate the member fails due to twisting. When the beam undergoes the Torsional behavior the formation of the crack is not limited to the one face, it propagates on all the faces which leads to the failure of section without further increase in the Torsional strength. The principle parameter which influence the Torsional strength were concrete compressive strength, however size of the beam, percentage of longitudinal and transverse reinforcement does not have much influence on the strength but they contribution towards post ductility of the beam.

Table 1: Analysis of Test Data (Experimental Data)

BEAM	T_u (expt) kN-m	$\tau_u = T_{uexp}/Bd$ Mpa	$T_{u\,(theory)}$ in kN-m								
			SKEW-BENDING THEORY	SPACE TRUSS ANALOGY	IS-456	AS3600	BS8110	ACI	European standards		CSA
									1	2	
MSC/2/1.0D	2.71	0.29	3.67	2.98	3.11	1.96	2.9	3.57	3.57	3.22	3.01
MSC/2/1.5D	2.47	0.27	2.88	1.99	2.07	1.31	1.94	2.38	2.38	3.22	2.01
MSC/2/2.0D	2.35	0.25	2.48	1.49	1.55	0.98	1.45	1.79	1.79	3.22	1.51
MSC/3/1.0D	3.01	0.33	3.67	2.98	3.11	1.96	2.9	3.57	3.57	4.83	3.01
MSC/3/1.5D	2.89	0.31	2.88	1.99	2.07	1.31	1.94	2.38	2.38	4.83	2.01
MSC/3/2.0D	2.71	0.29	2.48	1.49	1.55	0.98	1.45	1.79	1.79	4.83	1.51
MSC/4/1.0D	3.19	0.35	3.67	2.98	3.11	1.96	2.9	3.57	3.57	6.44	3.01
MSC/4/1.5D	2.77	0.3	2.88	1.99	2.07	1.31	1.94	2.38	2.38	6.44	2.01
MSC/4/2.0D	2.59	0.28	2.48	1.49	1.55	0.98	1.45	1.79	1.79	6.44	1.51
HSC/2/1.0D	3.44	0.37	3.82	2.98	3.11	1.96	2.9	3.57	3.57	3.22	3.01
HSC/2/1.5D	3.5	0.38	3.02	1.99	2.07	1.31	1.94	2.38	2.38	3.22	2.01
HSC/2/2.0D	3.44	0.37	2.63	1.49	1.55	0.98	1.45	1.79	1.79	3.22	1.51
HSC/3/1.0D	2.89	0.31	3.82	2.98	3.11	1.96	2.9	3.57	3.57	4.83	3.01
HSC/3/1.5D	3.25	0.35	3.02	1.99	2.07	1.31	1.94	2.38	2.38	4.83	2.01
HSC/3/2.0D	4.1	0.44	2.63	1.49	1.55	0.98	1.45	1.79	1.79	4.83	1.51
HSC/4/1.0D	3.5	0.38	3.82	2.98	3.11	1.96	2.9	3.57	3.57	6.44	3.01
HSC/4/1.5D	3.86	0.42	3.02	1.99	2.07	1.31	1.94	2.38	2.38	6.44	2.01
HSC/4/2.0D	3.92	0.42	2.63	1.49	1.55	0.98	1.45	1.79	1.79	6.44	1.51

Table 2: Ratio of Experimental to Theoretical Torsional Moment Strength

Beams	Skew bending theory	Space truss analogy	IS-456	AS3600	BS8110	ACI	Euro1	Euro2	CSA
MSC/2/1.0D	0.74	0.91	0.87	1.39	0.93	0.8	0.76	0.84	0.9
MSC/2/1.5D	0.86	1.24	1.19	1.89	1.28	1	1.04	0.77	1.23
MSC/2/2.0D	0.95	1.58	1.51	2.4	1.62	1.3	1.32	0.73	1.56
MSC/3/1.0D	0.82	1.01	0.97	1.54	1.04	0.8	0.84	0.62	1
MSC/3/1.5D	1.01	1.46	1.4	2.22	1.49	1.2	1.22	0.6	1.44
MSC/3/2.0D	1.09	1.82	1.75	2.77	1.87	1.5	1.52	0.56	1.8
MSC/4/1.0D	0.87	1.07	1.03	1.63	1.1	0.9	0.89	0.5	1.06
MSC/4/1.5D	0.96	1.4	1.34	2.12	1.43	1.2	1.16	0.43	1.38
MSC/4/2.0D	1.05	1.74	1.67	2.65	1.78	1.5	1.45	0.4	1.72
HSC/2/1.0D	0.9	1.15	1.11	1.75	1.18	1	0.96	1.07	1.14
HSC/2/1.5D	1.16	1.76	1.69	2.68	1.81	1.5	1.47	1.09	1.74
HSC/2/2.0D	1.31	2.31	2.21	3.51	2.37	1.9	1.92	1.07	2.28
HSC/3/1.0D	0.76	0.97	0.93	1.48	1	0.8	0.81	0.6	0.96
HSC/3/1.5D	1.08	1.64	1.57	2.49	1.68	1.4	1.37	0.67	1.62
HSC/3/2.0D	1.56	2.75	2.64	4.19	2.82	2.3	2.3	0.85	2.72
HSC/4/1.0D	0.92	1.17	1.13	1.79	1.2	1	0.98	0.54	1.16
HSC/4/1.5D	1.28	1.94	1.86	2.96	1.99	1.6	1.62	0.6	1.92
HSC/4/2.0D	1.49	2.63	2.52	4	2.7	2.2	2.2	0.61	2.6

EXPERIMENTAL AND LITERATURE DATA

The ratio of experimental Torsional moment and theoretical Torsional moment calculated by different theories and codes with, The average, standard deviation and coefficient of variation for experimental data and literature data were given in Table 3 and Table 4 respectively, we can see from table 3, that for experimental data the Canadian code predicts slightly better results. But it underestimates the Torsional strength. The skew bending theory predicts the value better than the other codes with a mean of 1.04, standard deviation of 0.24 and a coefficient of variation of 0.23.

From the Table 2, we can see that for literature data the **EUROPIAN CODE** predicts the value better than the other codes with a mean of 1.09, standard deviation of 0.68 and a coefficient of variation of 0.62. The skew bending theory overestimates the Torsional strength of the reinforced concrete section.

Table 3: Comparisons of Values Between Different Codes (Experimental Data)				Table 4: Comparisons of Values Between Different Codes (Literature Data) [7,8,11,12,14,15,16]			
CODES	MEAN	SD	CV	CODES	MEAN	SD	CV
SKEW BENDING THEORY	1.04	0.24	0.23	SKEW BENDING THEORY	1.90	0.74	0.39
SPACE TRUSS ANALOGY	1.59	0.55	0.35	SPACE TRUSS ANALOGY	1.43	0.75	0.53
IS 456	1.52	0.53	0.35	IS 456	1.50	0.79	0.53
AS 3600	2.41	0.84	0.35	AS 3600	1.64	0.81	0.49
BS 8110	1.63	0.57	0.35	BS 8110	1.35	0.65	0.48
ACI	1.32	0.46	0.35	ACI 318-05	1.85	1.06	0.57
EC 1	1.32	0.46	0.35	EC 1	1.3	0.69	0.53
EC 2	0.7	0.21	0.30	EC 2	1.09	0.68	0.62
CSA A23.3-94	1.57	0.54	0.35	CSA A23.3-94	1.3	0.62	0.48

Graphs has been plotted showing the variation of the predicted torsional strength by different codes with that of the experimental torsional moment from Figure 1 to Figure 8.

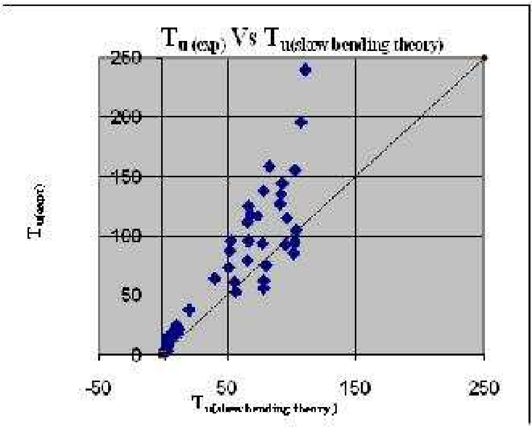


Figure 1: Graph $T_{u(exp)}$ v/s $T_{u(skew\ bending\ theory)}$

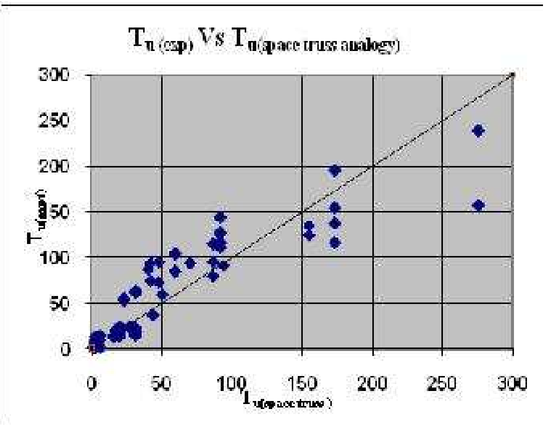


Figure 2: Graph $T_{u(exp)}$ v/s $T_{u(space\ truss)}$

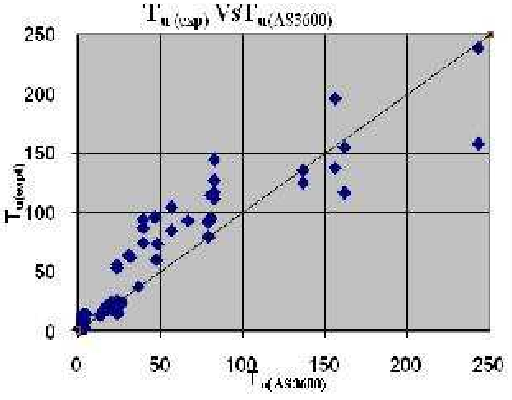


Figure 3: Graph $T_{u(exp)}$ v/s $T_{u(AS\ 3600)}$

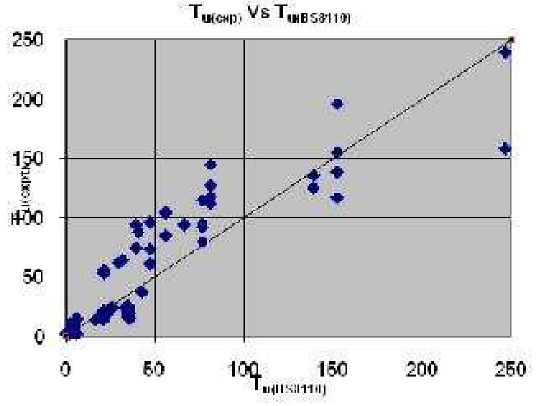


Figure 4: Graph $T_{u(exp)}$ v/s $T_{u(BS8110)}$

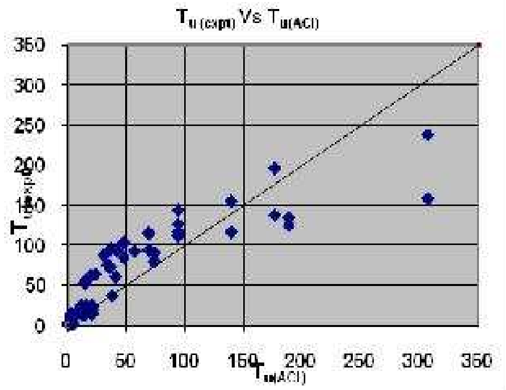


Figure 5: Graph $T_{u(exp)}$ v/s $T_{u(ACI)}$

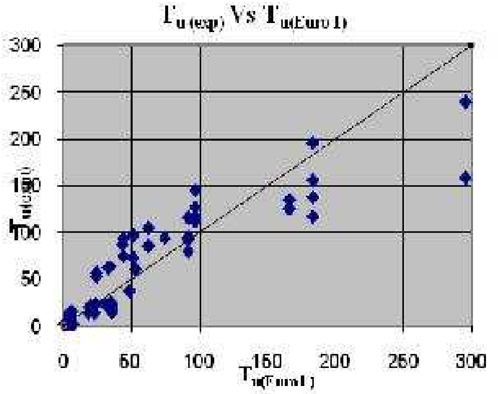


Figure 6: Graph $T_{u(exp)}$ v/s $T_{u(Euro\ 1)}$

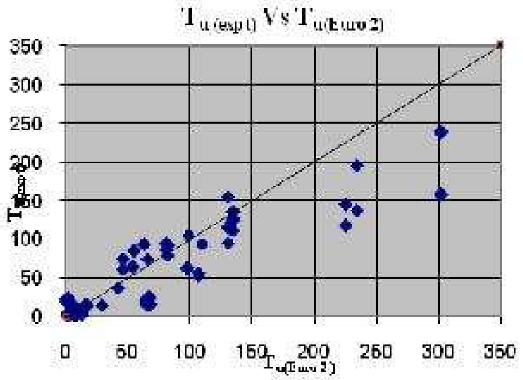


Figure 7: Graph $T_{u(exp)}$ v/s $T_{u(Euro\ 2)}$

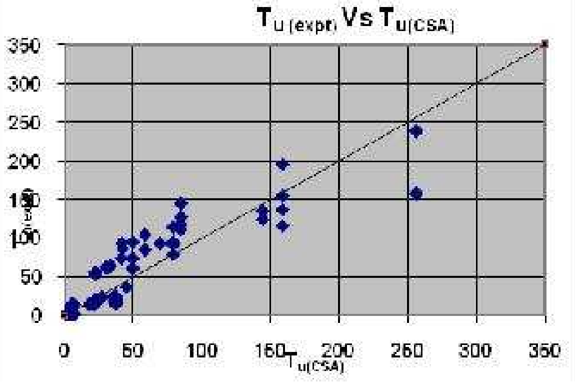


Figure 8: Graph $T_{u(exp)}$ v/s $T_{u(CSA)}$

CONCLUSIONS

1. HSC beams tested in the present investigations showed bursting type of failure at the ultimate Torsional moments. The effect of longitudinal reinforcement is not much as compared with the transverse reinforcement.
2. It was observed that as the compressive strength of concrete increases the Torsional strength of beam also increases. HSC provides higher Torsional strength than MSC for beams designed with the same amount of reinforcement.
3. In general, the HSC beams had the similar ductile behavior compared over MSC beams. The post peak strength decay was relatively steeper in HSC beams, especially for those with heavier reinforcement.
4. As the spacing of transverse reinforcement increases, the Torsional stress decreases for MSC beams, but the effect of transverse reinforcement was not clear in HSC beams. It has been observed that the Torsional shear stress increases with increase in the span to depth ratio.
5. It has been found that as the percentage of longitudinal reinforcement increases, the ultimate Torsional shear stress of all the beams increased.
6. It has been observed from the **Table 2**, that the skew bending theory predicts the value of Torsional strength much better than the other codes for the experimental beams.
7. The longitudinal reinforcement is not considered to calculate Torsional strength in codes and the theory, except European code, which predicted the values better when compared to other codes.
8. The variation of the fourth root of depth with the shear stress is shown in **Figure 5**. It can be seen from the graph that as the depth increases, there is an increase in the shear stresses of the beams. Fourth root of depth has significant effect on the Torsional strength.
9. For the present investigation ratio of the experimental Torsional moment to the predicted Torsional moment from skew bending theory was calculated and the mean, standard deviation and the coefficient of variation was found to be 1.04, 0.24 and 0.23 respectively and is shown in **Table 2**.

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