IASET: Journal of Applied Mathematics
& Statistical Sciences (IASET: JAMSS)
ISSN(P): Applied; ISSN(E): Applied
Vol. 1, Issue 2, Jul - Dec 2016; 15-20

PREDATOR-PREY INTERACTIONS WITH INFECTED AND SUSCEPTIBLE PREDATOR INCORPORATING HARVESTING OF PREDATOR

AHMED BUSERI ASHINE

Department of Mathematics, Madda Walabu University, Bale Robe, Ethiopia, East Africa

ABSTRACT

In this article, predator prey interactions where the predator is exposed to the risk of disease and harvesting is proposed. Equilibrium points, boundedness, and non-periodic solutions of themodelare obtained. Local stability and global stability were discussed. The equilibrium was stable locally, but not globally.

KEYWORDS: Prey-Predator, Stability, Harvesting, Dulac's Criterion

Subject Classification: 92B05

1. INTRODUCTION

Prey-predator models are of great interest to researchers in mathematics and ecology because they deal with environmental problems such as community's morbidity and how to control it, optimal harvest policy to sustain a community, and others. In the physical sciences, generic models can be constructed to explain a variety of phenomena. However, in the life sciences a model only describes a particular situation. Simple models such as the Lotka-Volterra are not able to tell us what is going on in the majority of cases. One of the reasons is due to the complexity of the biological ecosystem. Hence, we still seek for a variety of models to describe nature.

Theoretical and numerical studies of these models are able to give us an understanding of the interactions that is taking place. A particular class of models considers the existence of a disease in the predator or prey. Several models were constructed to study particular cases. To ensure the existence of the species involved, one of the steps taken is to harvest the infected species. In this paper, we consider the case where the infected predator is harvested. Several related theoretical studies have been conducted.

Amongst them are studies on the disease spread among the prey and the epidemic among predators with action incidence [6], the role of transmissible disease in the Holling Tanner predator prey model [4], the analysis of prey predator model with disease in the prey [7], another's study the disease in Lotka Volterra, [3] study the dynamics of a fisher resource system in an aquatic environment in two zones harvest in reserve area, [5] study the harvesting of infected prey, [1] show the stability analysis of harvesting, [2] Study the stability of harvested when the disease affects the predator by using the reproduction number.

The model is introduced after this section, followed by analysis on boundedness and properties of the solutions.

2. THE MATHEMATICAL MODEL

Consider the following dynamical system:

$$\begin{cases} \dfrac{dx}{dt} = a(1-x)x - b(y+z)x \\ \dfrac{dy}{dt} = cxy + \alpha yz - h_1 y \\ \dfrac{dz}{dt} = cxz - \alpha yz - h_2 z \end{cases} \quad (1)$$

where *x, y, z* are the prey, infected predator and susceptible predator respectively; α is the growth rate of prey; b , c the capture rate $(b > c)$, is the contact rate between the susceptible and infected predator; h_1 , and h_2 are the harvest rates of the infected and susceptible predator respectively, and assume that the less effective predator shall be easier to harvest, so; it is better also to assume infected predator not become susceptible again and finally the disease does not affect the ability of the infected predator attacking prey.

2.1. Bounded of Solutions

Theorem 1

The solution of system (1) is bounded.

Proof

Let the function $w(x, y, z) = x(t) + y(t) + z(t)$ and let μ be a positive number such that $0 < \eta < h_2$.

Then, $w'(t) + uw = ax(1-x) + \eta x - (b-c)(y+z)x - (h_1 - \eta)y - (h_2 - \eta)z$

$$w'(t) + uw < -a\left(x^2 - \left(\frac{a+\eta}{a}\right)x + \left(\frac{a+\eta}{2a}\right)^2 \right) + \frac{1}{a}\left(\frac{a+\eta}{2}\right)^2$$

Let $\frac{1}{a}\left(\frac{a+\eta}{2}\right)^2 = v$

$$w'(t) + uw(t) \le v$$

$$0 < w(x, y, z) \le \frac{v}{u}\left(1 - e^{-ut}\right) + e^{-ut}(x, y, z)\big|_{t \to 0}$$

Theorem 2

Let $F(x, y) = ax - ax^2 - bxy$, $G(x, y) = cxy - h_1 y$,

$M(x, z) = ax - ax^2 - bxz$, $N(x, z) = cxz - h_2 z$.

Define a function $H(x,y)=\dfrac{1}{xz}$.

Then $Q(x,y)=\dfrac{\partial(HF)}{\partial x}+\dfrac{\partial(HG)}{\partial y}=-\dfrac{a}{y}$

It's clear that is no change in change sign; therefore, this system cannot have any periodic solution in the xy-plane.

Again, $Q(x,z)=\dfrac{\partial(HM)}{\partial x}+\dfrac{\partial(HN)}{\partial z}=-\dfrac{a}{z}$

There is no change in sign; therefore, there is no periodic solution in xz- plane.

Hence, the system has no periodic solution.

2.2. Equilibrium

The dynamical system (1) has the following five fixed points: the origin (E_1), a predator fee fixed point (E_2), a disease free fixed point (E_3), a fixed point when all predator infected (E_4), and a fixed for which both population survive (E_5):

$$E_1:(x,y,z)=(0,0,0)$$

$$E_2:(x,y,z)=(1,0,0)$$

$$E_3:(x,y,z)=(x_2,0,z_2)\text{; where } x_2=\frac{h_2}{c}\text{ , } z_2=\frac{a(1-x)}{b}$$

$$E_4:(x,y,z)=(x_3,y_3,0)\text{; where } x_3=\frac{h_1}{c}\text{ , } y_3=\frac{a(1-x)}{b}$$

$$E_5:(x^*,y^*,z^*)=\left(1-\frac{b}{a\alpha}(h_1-h_2),\frac{cx^*-h_2}{\alpha},\frac{h_1-bx^*}{\alpha}\right)$$

3. STABILITY

The Jacobian matrix of system (1) is given by:

$$J(x,y,z)=\begin{pmatrix} a-2ax-b(y+z) & -bx & -bx \\ cy & cx+\alpha z-h_1 & \alpha y \\ cz & -\alpha z & cx-\alpha y-h_2 \end{pmatrix}$$

Case 1: The System without Disease

When infected predators eradicate, the system (1) becomes:

$$\begin{cases} \dfrac{dx}{dt} = a(1-x)x - bxz \\ \dfrac{dz}{dt} = cxz - h_2 z \end{cases} \quad \textbf{(1a)}$$

The equilibrium (nontrivial) are $E_c'(1,0)$, $E_c'(x_2, z_2)$ where, $x_2 = \dfrac{h_2}{c}, z_2 = \dfrac{a}{b}(1-x_2)$

Proposition 1

$E_c'(1,0)$ is stable when $h_2 > c$ and unstable otherwise.

Proof

The eigenvalues near the first equilibrium are $-a$ and $c - h_2$. This completes the proof.

Theorem 3

If the equilibrium $E_c'(x_2, z_2)$ is locally stable, then the basin of attraction of this

equilibrium is denoted by $B(E_c'(x_2, z_2))$,

where $B(E_2') = \{(x,z) \in \Re_+{}^2 : x > \dfrac{h_2}{c}, z > \dfrac{a}{b}(1-x) with \dfrac{h_2}{c} z < \dfrac{a}{b}(1-x)x\}$

Proof

Let $V(x,z)$ be a function where

$V(x,z) = \left(x - x_2 - x_2 \log^{\frac{x}{x_2}} \right) + \left(z - z_2 - z_2 \log^{\frac{z}{z_2}} \right)$, the

$$\frac{dV}{dt} = -a(x-x_2)^2 - (b-c)(x-x_2)(z-z_2) < 0$$

Remark:

The eigenvalues near $E_2'(x_2, z_2)$ are $\dfrac{-ax_2}{2} \pm \dfrac{\sqrt{a^2 x_2{}^2 - 4ah_2(1-x_2)}}{2}$ and

$h_2 + \alpha z_2 - h_1$, and stable when $1 - \dfrac{a\alpha(1-x_2)}{b(h_1 - h_2)} > 0$

Case 2: When all Predators Become Infected

When all predators become infected the subsystem of system (1) becomes:

$$\begin{cases} \dfrac{dx}{dt} = a(1-x)x - bxy \\ \dfrac{dy}{dt} = cxy - h_1 y \end{cases} \quad \textbf{(1b)}$$

The equilibrium (nontrivial) are $E_c'(1,0)$, $E_c'(x_3, y_3)$ where, $x_3 = \dfrac{h_1}{c}$, $y_3 = \dfrac{a}{b}(1-x_3)$

Proposition 2

$E_c'(1,0)$ is stable when $h_1 > c$ and unstable otherwise.

Proof

The eigenvalues near $E_c'(1,0)$ are $-a$ and $c - h_1$. This completes the proof.

Theorem 4

Assume the equilibrium $E_c'(x_3, y_3)$ is locally stable, the basin of attraction of this equilibrium is denoted by $B(E_c'(x_3, y_3))$ where $B(E_c') = \{(x,z) \in \Re_+^{\,2} : x > x_3, y > y_3\}$

Proof

The proof is the same as in theorem (3).

Proposition 3

The equilibrium $E_c'(x_3, y_3, 0)$ is stable with condition $0 > 1 - \dfrac{a\alpha(1-x_3)}{b(h_1 - h_2)}$

Proposition 4

The stability near the equilibrium $E_c^*(x^*, y^*, z^*)$ is given by equation

$\lambda^3 + A\lambda^2 + B\lambda + C = 0$ where

$A = ax^* > 0$, $B = bx^*c(y^* + z^*) + \alpha^2 y^* z^*$, $C = a\alpha^2 x^* y^* z^* > 0$

$AB - C - = ax^*(bx^*c(y^* + z^*)) > 0$

From Routh-Hurwitz stability criterion it is stable.

4. CONCLUSIONS

In this paper, the discussion and analysis model prey predator interaction with harvesting of predator is presented. Boundedness of solution, and equilibriums points with their conditions were discussed. The basin attraction of some of equilibrium points was also calculated. Finally, the result shows us the infected predator increases while the susceptible predator decreasing.

REFERENCES

1. Azar, C., J. Holmberg, et al. Stability analysis of harvesting in a predator-prey model. Journal of Theoretical Biology., 174(1) (1995), 13-19.
2. Chevé, M., R. Congar, et al. (2010). Resilience and stability of harvested predator-prey systems to infectious diseases in the predator.
3. Dubey, B., P. Chandra, et al. A model for fishery resource with reserve area. Nonlinear Analysis: Real World Applications., 4(4)(2003), 625-637.
4. Haque, M. and E. Venturino. The role of transmissible diseases in the Holling–Tanner predator–prey model. Theoretical Population Biology., 70(3)(2006), 273-288.
5. S. A. Wuhaib, Y. AbuHasan, (2012). Apredator Infected Prey Model with Harvesting of Infected Prey .ICCEMS2012: 59-63
6. Venturino, E. Epidemics in predator–prey models: disease in the predators. Mathematical Medicine and Biology., 19(3)(2001), 185-205.
7. Y.X, L. Chen. Modelling and analysis of a predator prey model with disease in the prey. Mathematical Biosciences., 171(2001), 59-82.