

## MATHEMATICAL ESTIMATION OF PLANT POPULATION

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### ABSTRACT

The yield rate of a cropping area depends on the spacing of plants because adequate number of plants can only support to enhance the rate of production. The more number of plants uses more amount of water through transpiration for which crop failure is a common phenomenon in rain fed areas. The area under study is a rectangular plot which is partitioned into finite number of circular seed beds of same sizes. The lost regions formed on the side of the circular seed beds are asteroids of same sizes. The non linear programming problem is formulated to decide the optimum number of plants for different types of crops. Lagrange's method is used to solve the formulated optimization problem. Though the minimum numbers of plants are estimated, still then it is accepted because of more plant spacing which can produce more individual rate of production of each plant. Finally the numbers of plants estimated and number of plants observed for different crops are compared to decide the better strategy which can enhance the rate of production without the effect of associated crop parameters like pattern of land, manure and seed quality respectively.

**KEYWORDS:** Asteroids, Lagrange's Method, Circular Seed Bed, Hessian Bordered Matrix

### INTRODUCTION

The economic development of a locality depends on the net return from its agricultural productivities. To ensure high yield rate, the proper care of unavoidable parameters such as manure, water availability, land type, quality of seed and plant spacing are essential to boost the productivity of the farming community. Among all factors plant spacing plays a vital role to enhance the rate of production. The yield rate of a cropland depends on the population of plants. Under low plant population though yield of individual plant increases due to large spacing among plants, the sum amount of yield decreases due to availability of less number of plants and vice versa. The yield rate per plant increases up to certain level of population. This optimum number of plants can support the decision maker to enhance the level of production without the effect of the associated crop parameters. The increasing number of plant population in the rain fed areas can consume even more amount of water through transpiration; as a consequence crop failure is a seasonal trend. Therefore number of plants should be less than average for the rain fed cropping areas. Estimation of accurate number of plants for a cropping area is a measure parameter for increasing the net agricultural of production.

### STUDY OF LITERATURE

The yield rate of a crop land is associated with several numbers of inputs. Among all factors the optimum number of plant population can support the decision maker to enhance the rate of production without affecting key parameters. The accurate estimation of plant population can project the output of the cropping area. Several authors worked to estimate the optimum number of plants of a cropping area such as (O.C Adebooye etl. 2006) used non linear programming plant population model to estimate the number of plants. The model was consisting of rectangular seed beds for analysing the outcomes of the crop model. The generated optimal solution was compared with the observed number of plants.

(Raouf Seyed etl 2009) introduced factorial experiment on maize hybrids to estimate the number of plants which was based on randomized complete block design. Further it was analysed by(H. Rasekh etl. 2010) to know the effect of planting pattern and plant density on physiological characteristics and yield rate by using randomized complete block design for three successive replications. They showed square planting has higher rate of return than rectangular pattern. In real situation the nature of each crop land is not square type. As a consequence above analysed methods of estimating the plant population cannot be accepted for optimizing the net return from agriculture sources.

## MATERIALS AND METHODS

The plant population of a cropping area can be estimated by partitioning the crop land into circular areas called seed bed of constant radius. The plant population (Pp) is represented as

$$Pp = \frac{L*B*N}{\pi*a^2} \quad (1)$$

Where length is  $L$  and breadth is  $B$  of the crop land. Let  $a$  be the radius of each partitioned circular shed bed and ' $N$ ' be the number of baby plants available in each circular shed. Now the area of each circular seed bed is  $\pi a^2$  which is fixed for the defined crop land.

$$\text{Therefore } \pi a^2 = k_1$$

$$\text{In other words } -\pi a^2 + k_1 = 0$$

There is a discrepancy between estimated plant population using (1) and the observed plant population which is due to loss of plants in four adjacent neighbouring circular seed beds. The region generated at the intersection of four neighbouring sheds structured is like an asteroide. The polar form of representation of each asteroide is

$$x = b(\cos\theta)^3 \text{ And } y = b(\sin\theta)^3 \text{ where } 0 \leq \theta \leq 2\pi$$

The area of each asteroide is  $\frac{3\pi b^2}{8}$  where  $2b$  is length of diagonal of each asteroide. since the seed beds of the cropping area are equal, in the loss region the area of the generated asteroids are fixed. That is  $\frac{3\pi b^2}{8} = k_2$  where  $k_2$  is area of each asteroide. Since one asteroide is formed for four numbers of seed beds, the total number of asteroids estimated must be  $\frac{N}{4}$

$$\text{The amount of area left} = \frac{3\pi b^2}{8} * \frac{N}{4} = \frac{3\pi N b^2}{32} \quad (2)$$

Further the area of the residue region on the boundary of the crop land is consisting of semi asteroids  $\frac{3\pi N b^2 (L+B)}{8}$

$$\text{Therefore the sum total residue area designed for plantation} = \frac{3\pi N b^2}{32} + \frac{3\pi N b^2 (L+B)}{8}$$

$$\text{The total population (Pp) of plant} = \frac{L*B*N}{\pi*a^2} + \frac{3\pi N b^2}{32} + \frac{3\pi N b^2 (L+B)}{8} \quad (3)$$

But the above population varies with respect to dimension of seed bed and gap between the baby plants. The aim of the present problem is to optimize the plant population subject to the fixed area constraint of each seed bed. The mathematical formulation of the non linear problem is Optimize (Pp) =  $\frac{L*B*N}{\pi*a^2} + \frac{3\pi N b^2}{32} + \frac{3\pi N b^2 (L+B)}{8}$

**Subject to**

**Constraints**

$$-\pi a^2 + k_1 = 0$$

$$\frac{3\pi b^2}{8} - k_2 = 0$$

$$a \geq 0, b \geq 0$$

The Lagrange's function ( $L_f$ ) can be formed by using [4] as

$$L_f = Pp - \mu_1(k_1 - \pi a^2) - \mu_2\left(\frac{3\pi b^2}{8} - k_2\right)$$

$$\Rightarrow L_f = \frac{L*B*N}{\pi*a^2} + \frac{3\pi N b^2}{32} + \frac{3\pi N b^2}{8}(L + B) - \mu_1(k_1 - \pi a^2) - \mu_2\left(\frac{3\pi b^2}{8} - k_2\right)$$

The necessary conditions for stationary points are obtained by using the following relations

$$\frac{\partial L_f}{\partial a} = -2 * \frac{L*B*N}{\pi*a^3} + \mu_1 * 2\pi a = 0 \tag{4}$$

$$\frac{\partial L_f}{\partial b} = \frac{\pi*N*B}{16} - \frac{3\pi b}{4} \mu_2 = 0 \tag{5}$$

$$\frac{\partial L_f}{\partial \mu_1} = -(k_1 - \pi a^2) \tag{6}$$

$$\text{and } \frac{\partial L_f}{\partial \mu_2} = \frac{3\pi b^2}{8} - k_2 = 0 \tag{7}$$

Solving equations (4), (5), (6) and (7) simultaneously the stationary point is

$$a = \sqrt{\frac{k_1}{\pi}}, b = \sqrt{\frac{8k_2}{3\pi}} \mu_1 = \frac{L*B*N}{k_1^2} \text{ and } \mu_2 = \left(\frac{1}{4} + L + B\right). \text{Where } \mu_1 \text{ and } \mu_2 \text{ are Lagrange's multipliers used}$$

to estimate the optimum value of the non linear programming problem. The generated stationary point may be a maximum or a minimum point of the objective function that can be tested using the Hessian Bordered matrix. Now the Hessian Bordered matrix ( $H_B$ ) of the optimization problem is

$$H_B = \begin{bmatrix} 0 & 0 & \frac{\partial g_1}{\partial a} & \frac{\partial g_1}{\partial b} \\ 0 & 0 & \frac{\partial g_2}{\partial a} & \frac{\partial g_2}{\partial b} \\ \frac{\partial g_1}{\partial a} & \frac{\partial g_2}{\partial a} & \frac{\partial^2 L}{\partial a^2} & \frac{\partial^2 L}{\partial a \partial b} \\ \frac{\partial g_1}{\partial b} & \frac{\partial g_2}{\partial b} & \frac{\partial^2 L}{\partial b \partial a} & \frac{\partial^2 L}{\partial b^2} \end{bmatrix}$$

The value of the ( $H_B$ ) determinant is

$$|H_B| = \begin{vmatrix} 0 & 0 & -2\pi a & 0 \\ 0 & 0 & 0 & \frac{3\pi b}{4} \\ -2\pi a & 0 & 2\pi\mu + \frac{6LBN}{a^4} & 0 \\ 0 & \frac{3\pi b}{4} & 0 & \frac{3\pi Nb}{16} + \frac{3\pi N(L+B)}{4} + \frac{3\pi\mu}{4} \end{vmatrix} = \frac{9}{4}(ab^2)^*\pi^4 > 0$$

Since value of Hessian Bordered matrix is positive

The stationary point  $(a, b)$  of the non linear programming problem is verified as a minimum point of the objective function and the optimum number of number of estimated plants  $Pp = \left[ \frac{L*B}{k_1} + \frac{k_2}{4} (4L + 4B + 1) \right] * N$ .

Where  $k_1$  is area of each uniform circular seed bed and  $k_2$  is the area of each asteroid of the cropland as shown in figure (1.1)

Result and discussion: Though several methods are available for seed bed cultivation, circular seed bed is useful for the farming community because the shortest surface area for making garden is more productive. This seeding process also helps microorganisms and bacteria by fixing nutrients for the plants.

The analysis of plant population as given in table 1.1 indicates mathematically estimated number of plants is more than the traditional practice

**Table 1.1: Comparative Analysis of Plant Population by Physical Counting and Optimization Method**

Name of Plants	Inter Plant Spacing in Square Inch	Physical Counting of Number of Plants in 75 Square Feet Crop Land	Mathematically Estimated Number of Plants in 75 Square Feet Crop Land	Difference %
Marigold(Dwarf)	12X12	91	86	5
Dahlia(Small)	12X12	93	86	7
Salvia	15X15	64	55	14
Pumpkin	6X6	368	346	5
Petunia	12X12	92	86	6

**CONCLUSIONS**

Plant spacing plays a vital role in the field of agriculture. In the irrigated areas more number of plants can yield more amounts of production without the involvement of agricultural parameters but in the case of rain fed regions the more number of plants wastes more water through transpiration which causes crop failure and drought. Therefore the plant density determination is a minimized problem and the estimated number of plants so determined can support the decision maker to yield high return. For different crops the numbers of estimated plants are not same. The comparative analysis of crop density indicates the number of plants estimated by optimization method is less than the traditional method.

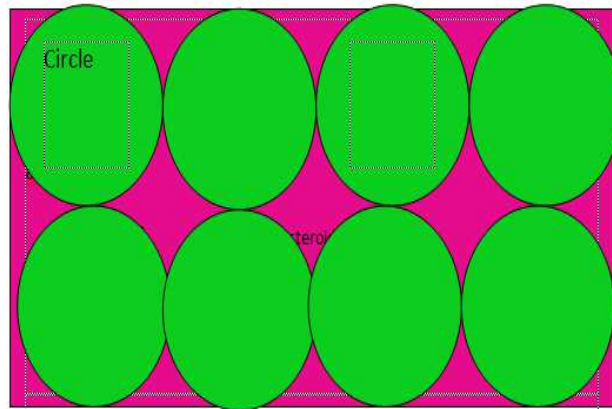
**REFERENCES**

1. .C Adebooye, S O Ajadi and A. B Fa qbohum(2006) An accurate mathematical formula for estimating plant population in a four dimensional field of sole crop, Science Alert, journal of Agronomy,5,289-292.
2. Raouf Seyed Sharifi, Mohammad Sedghi and Abdolghayoum Gholipouri (2009) Effect of population density on

yield and yield attributes of maize hybrids, Volume 4, Issue- 4, page 375-379

3. H .Rasekh, J. A Saghari, M.N Safarzadeh Wishkai, S L Massoum and R.Zakerinejad ( 2010)Effect of planting pattern and plant density on physiological characteristics and yield of peanut in Iran. Research Journal of Biological Science, Volume 5, Issue 8 (542-547).
4. Research Journal of Biological Science, Volume 5, Issue 8 (542-547).
5. S D Sharma (2004) Operations Research, Kedarnath Ramnath &Co Publisher Meerut, Fourteenth Edition.

## APPENDICES



**1.1 Circular Seed Bed Garden**

**Figure 1.1: The Circular Seeding Bed by Partitioning**

